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ARTICLES

IS THERE PROPOSITIONAL UNDERSTANDING?

Emma C. GORDON

ABSTRACT: Literature in epistemology tends to suppose that there are three main types of understanding – propositional, atomistic, and objectual. By showing that all apparent instances of propositional understanding can be more plausibly explained as featuring one of several other epistemic states, this paper argues that talk of propositional understanding is unhelpful and misleading. The upshot is that epistemologists can do without the notion of propositional understanding.

KEYWORDS: epistemology, understanding, knowledge, propositional understanding

1. Introduction

Understanding is a kind of cognitive achievement of which the object is "strikingly varied."[1] For example, we make claims to understand the psychology of loved ones, the workings of machines, current events, the structure of languages, and academic hypotheses. However, in spite of the extent to which we obviously strive to understand, the epistemic state of understanding has remained an under-discussed topic in epistemology, and usage of the term is often surprisingly ambiguous. As Zagzebski observes, different uses of 'understand' seem to mean so many different things that it is difficult to even pick out the precise state that has been ignored, and this can lead to a vicious circle – in other words, "neglect leads to fragmentation of meaning, which seems to justify further neglect and further fragmentation until eventually a concept can disappear entirely."[2] However, it is important that more efforts be made to remedy this. The disproportionate attention devoted to knowledge in particular is rather troubling when we consider that there are various compelling motivations for thinking that understanding seems just as valuable as knowledge (if not *more* valuable[3] than knowledge). Riggs

[1] Stephen Grimm, "Understanding," in *Routledge Companion to Epistemology*, eds. Sven Bernecker and Duncan Pritchard (New York: Routledge, 2010).

[2] Linda Zagzebski, *On Epistemology* (Belmont: Wadsworth, 2009), 141.

[3] For some observations and arguments to this effect, see Jonathan Kvanvig, *The Value of Knowledge and the Pursuit of Understanding* (New York: Cambridge University Press, 2003), as well as Duncan Pritchard, "Knowledge, Understanding and Epistemic Value," in *Epistemology (Royal Institute of Philosophy Lectures)*, ed. Anthony O'Hear (Cambridge:

asks "Why the longstanding bias in favour of knowledge, justification and the like at expense of understanding?" and adds "I suspect that at least one reason is that understanding is a harder phenomenon to account for and describe precisely than the aforementioned others."[4] He is quite right about the enormity of such a task – there is more than one type of understanding, and there is no doubt that offering an account of any of these types is a challenging project.

One vital early stage of finding out more about the phenomenon of understanding will involve investigating what sort of conditions must be fulfilled in order for one to understand. Given that it is highly plausible that more than one sort of understanding is relevant to epistemology, preliminary explorations of understanding will also contrast the conditions one must meet to attain different *sorts* of understanding. My particular goal herein is to supply good reasons for us to set aside one certain alleged sort of understanding in such future epistemological investigations. I submit that the notion of *propositional understanding* is misleading, and that if it is allowed to play a substantial role in theorising about understanding then it is capable of muddying the waters of more substantive and significant topics concerning understanding (such as whether, and to what extent, it might constitute a more significant cognitive achievement than does any kind of knowledge).

To begin, I will briefly review the main types of understanding that can be found in contemporary epistemological literature. I will then move on to focus specifically on propositional understanding, trying to better define what is meant by the term when it is employed. Next, I will contend that what might seem to be instances of propositional understanding can more plausibly be explained as featuring one of a group of importantly different (but closely related) epistemic states. In showing this, I will support my view in part by appealing to considerations about the conditions under which, when pressed, we will tend to quickly retract these sorts of apparent attributions of propositional understanding.

Cambridge University Press, 2009) and Dennis Whitcomb, "Epistemic Value," in *The Continuum Companion to Epistemology*, ed. Andrew Cullison (New York: Continuum Publishing Corporation, 2011).

[4] Wayne Riggs, "Understanding 'Virtue' and the Virtue of Understanding," in *Intellectual Virtue Perspectives from Ethics and Epistemology*, eds. Michael DePaul and Linda Zagzebski (Oxford: Oxford University Press, 2003), 19-20.

2. Attempting to define propositional understanding

Epistemologists interested in understanding often compare and contrast understanding with propositional knowledge.[5] If this project is to be undertaken, it makes sense to tackle the question of what properly distinguishes between different *sorts* of understanding rather early on in any investigation of understanding, in order to determine which one or more of these types is most likely to yield interesting results if compared with knowledge. The main types of understanding that we can draw from epistemological literature are as follows:[6]

> *Propositional understanding* or *understanding-that*: "I understand *that* X," e.g. "Andy understands that the meeting will be at 3pm."
>
> *Atomistic understanding* or *understanding-wh*: "I understand *why/when/where/what* X," e.g. "Lauren understands why the building is closing down."
>
> *Objectual understanding* or *holistic understanding*: "I *understand* X," e.g. "Mark understands human biology."[7]

Of these types, Pritchard[8] thinks that the paradigmatic sort of usage will concern atomistic understanding, such as "I understand why the house burned down" or "I understand why Johnny is behaving in this way." Objectual understanding, meanwhile, is the sort that Kvanvig awards primary focus to, describing it as obtaining "when understanding grammatically is followed by an object/subject matter, as in understanding the presidency, or the president, or

---

[5] I will not justify this commonplace methodology here, given that my goal is just to expose the idea of propositional understanding as an unhelpful distraction from the philosophically interesting concept of understanding.

[6] Different types of understanding are more prominent in other areas of philosophy. For example, linguistic understanding deals with what it means to understand words. For some prominent and recent work in this area, see Harriet E. Baber, "In Defence of Proselytizing," *Religious Studies* 36 (2003): 333-44 and Guy Longworth, "Linguistic Understanding and Knowledge," *Nous* 42 (2008): 50-79.

[7] All of these types of understanding are discussed in Kvanvig, *The Value of Knowledge*. For another discussion of propositional understanding, see Berit Brogaard, "I Know. Therefore, I Understand", unpublished draft (2005), https://sites.google.com/site/brogaardb/brogaard knowledgeunderstanding.pdf?attredirects=0 (accessed May 8, 2012). For work on atomistic understanding, see also Grimm, "Understanding," and Alison Hills, "Moral Testimony and Moral Epistemology," *Ethics* 120 (2009): 94-127. Further thoughts on the nature of objectual understanding can also be found in Grimm, and in Catherine Elgin, "Is Understanding Factive?" in *Epistemic Value*, eds. Duncan Pritchard, Alan Millar and Adrian Haddock (Oxford: Oxford University Press, 2009).

[8] See for example Pritchard, "Knowledge, Understanding."

politics."[9] However, since atomistic and objectual understanding are clearly worthy of much sustained attention, it is mainly propositional understanding with which I will be concerned herein – specifically, my focus will be on the question of whether there is some one epistemic state such that it is helpful for epistemologists to refer to this state as propositional understanding.

When thinking about the idea of propositional understanding, we might first wonder exactly what 'proposition' means in the context of discussing understanding as an epistemic state. As McGrath notes, the term is used throughout philosophical literature to refer to a rather wide variety of things – "the primary bearers of truth value, the objects of belief and other 'propositional attitudes' (i.e., what is believed, doubted, etc.), the referents of that-clauses, and the meanings of sentences."[10] I think it is fair to say that the sort of work with which we are currently concerned treats propositions as the objects of propositional attitudes and the referents of that-clauses, and I will hereafter assume that this is the case. With this small preliminary issue addressed, we can move on to ask what precisely has been said in the aid of defining propositional understanding and the conditions under which we might come to have it.

Kvanvig first describes propositional understanding as obtaining "when we attribute understanding in the form of a propositional operator, as in understanding that something is the case."[11] Similarly, Brogaard describes ascriptions of propositional understanding as being "ascriptions of understanding of something being the case."[12] However, Pritchard observes that understanding (unlike knowledge) at least isn't normally directly concerned with one proposition[13], and Kvanvig later supports the idea that such understanding is not particularly common when he says that "understanding has as its standard object a body of information, but ordinarily not a single proposition" and also states that there are "no single proposition of which we ascribe understanding"[14] when we claim that someone understands a subject matter.

We can begin to get a clearer picture of what the epistemic state of propositional understanding is supposed to be when we look more closely at why those who believe there *is* such a thing as propositional understanding might share

[9] Kvanvig, *The Value of Knowledge*, 191.

[10] Matthew McGrath, "Propositions," in *Stanford Encylopedia of Philosophy (Fall 2011 Edition)*, ed. Edward N. Zalta (2011), 1, URL= < http://www.seop.leeds.ac.uk/archives/fall2011/entries/propositions/>).

[11] Kvanvig, *The Value of Knowledge,* 191.

[12] Brogaard, "I Know," 2.

[13] Pritchard, "Knowledge, Understanding," 11.

[14] Kvanvig, *The Value of Knowledge,* 195.

the view that genuine instances of this sort of understanding are quite rare. In contrast to our highly commonplace ascriptions of propositional knowledge, consider that (purely comparatively speaking) it is not really all that often that we utter or hear sentences of the form "I understand that X" in conversation. Notice that this rarity seems particularly explicable when we consider that, at least frequently, sentences of this form actually just represent propositional knowledge. The sentence "it is time for dinner," for example, seems to almost always be used to express the same notion regardless of whether preceded by 'I understand that' or 'I know that.' The same is true of many other statements, such as "we are leaving at four o'clock" or "this is where Peter lives."[15] As Grimm[16] notes, most cases of "S understands that p" can be easily replaced by "S knows that p" without loss of meaning. Further, it seems that when uttering such sentences, we would generally be more likely to choose 'know that' rather than 'understand that' to precede them, especially if explicitly given the choice.

So, we can now see that perhaps propositional understanding is not a common epistemic state because 'knows that' and 'understands that' are often readily interchangeable, and in the main seem to represent propositional knowledge rather than something we would want to insist should be called genuine understanding. However, this leads us to wonder the following: when propositional understanding *does* occur in its alleged true form, what is it that in those particular cases distinguishes it from propositional knowledge? Kvanvig claims that these authentic, rare instances of the propositional form of understanding differ from propositional knowledge in that knowledge doesn't demand that the agent grasp or appreciate the explanatory relations between the items in a body of information. This grasp is commonly thought to be necessary when it comes to objectual and atomistic understanding. For example, Riggs states that understanding of a subject matter "requires a deep appreciation, grasp or awareness of how its parts fit together, what role each one plays in the context of the whole, and of the role it plays in the larger scheme of things."[17] In later work, Kvanvig[18] slightly expands on his original idea to claim that such relationships are so integral to understanding that any time we think about the nature of *any* kind

[15] Some apparently obvious examples like these will later be called into question, but for the moment all that matters is that such sentences do not strike us as obviously importantly different in *most* cases (whether preceded by 'I know that' or 'I understand that').

[16] Grimm, "Understanding," 3.

[17] Riggs, "Understanding 'Virtue,'" 19.

[18] See for example Kvanvig, "Assertion, Knowledge and Lotteries", in *Williamson on Knowledge*, eds. Duncan Pritchard and Patrick Greenough (Oxford: Oxford University Press, 2009), 140-60.

of understanding we immediately think of precisely these sorts of relationships, i.e. "the ways in which pieces of information are connected with each other."[19]

Now, it is not immediately obvious that propositional understanding should require this grasp of relations in the same way that atomistic and objectual understanding so plausibly do. However, one way in which Kvanvig suggests that propositional understanding could require grasping these kinds of relations is to say that such understanding might "result via abstraction" from what could plausibly be thought to be the primary form (i.e. understanding of a subject matter). On this view, then, it seems that if you are to have propositional understanding, you must understand a subject matter that includes the relevant proposition. For example, if a police officer says "I understand that Jimmy used a knife to commit the murder," he counts as having propositional understanding only if he also understands the relevant subject matter (perhaps that of this particular murder, or the particular crime scene at which the body was found), and if Wendy says "I understand that you won't be at the celebratory barbeque," she has propositional understanding only if she also understands the subject matter of your summer plans (or something along those lines). Similarly, if John says "I understand that red peppers are added at this point in the curry recipe," he only counts as having propositional understanding if he understands the relevant recipe, or (say) Thai cooking.

This seems to be the only way to make sense of Kvanvig's plausible idea that propositional understanding requires grasping coherence-making relations, but given that it requires one to have *objectual understanding* then it is not obviously a picture of a type of understanding that actually takes a *proposition* as its object (especially since Kvanvig also explicitly says that no understanding of singular propositions is ascribed when objectual understanding is ascribed). As such, it appears unhelpful for us to even call what the view describes 'propositional understanding' (as opposed to, say, calling this description merely an account of one of the things that people with *objectual* understanding can do).

In addition to not endorsing this specific idea of what propositional understanding would be (for the reasons just mentioned), I do not think there is

[19] The notion of these relationships, which Kvanvig calls "explanatory and other coherence-making relationships" is complicated and tough to explicate (to the extent that a full account is still lacking in the current literature). Even coming close to offering a theory of what grasping involves and what precisely must *be* grasped requires giving at least partial answers to a long chain of interrelated questions. For some work on what the act of grasping coherence-making relationships involves, and on what exactly is grasped when one understands, see once again Grimm, "Understanding," as well as Hills, "Moral Testimony," and Michael Strevens, "No Understanding Without Explanation," *Studies in History and Philosophy of Science* (2011 draft).

any obvious alternative account of propositional understanding that does not just collapse into an account of some other epistemic state. Rather, it is my position that 'understanding' that takes one proposition as its object is not a distinct breed of understanding at all. The notion of a type of understanding that *does* take one proposition as its object is a philosophically uninteresting (as well as misleading) construal of what it is to understand. In order to more convincingly show this, however, I must deal with several types of problem case in which it at first quite strongly appears that there is such a thing as propositional understanding.

3. Propositional knowing

Let's begin by looking more closely at cases that turn out to only feature propositional *knowledge* even though some might appear at first glance as though they feature genuine propositional understanding. This will help us figure out how to diagnose such cases more readily, and tell us something about how to set them apart from other cases of apparent propositional understanding.

Now, we saw at the outset that many apparent attributions of propositional understanding seem to merely be attributions of propositional knowledge, given that most sentences of the form "S understands that P" can be changed to "S knows that P" without any loss of meaning. Keeping this idea in mind, consider that Brogaard argues against Kvanvig's view that propositional understanding demands a grasp of coherence-making relationships by saying that you can assert your understanding that your flight was cancelled "without appreciating any explanatory or coherence-inducing relations in a larger body of information."[20] I agree she is quite right that no grasp of coherence-making relations is required in this particular case, but I think that this is because such a use of 'understand' is actually also one of the uses that are synonymous with 'know.' Presumably, all that the agent is trying to ascribe to himself is something along the lines of a strongly justified belief that his flight has been cancelled. This case does not constitute a convincing counterexample to Kvanvig's view, as it is simply a case of propositional knowledge (and, as we saw in section two, it is not at all clear what would demarcate propositional knowledge from propositional understanding were it possible to have the latter *without* also having some further sort of

[20] See Brogaard, "I Know," 6. Given that Brogaard thinks that propositional understanding *doesn't* require grasping coherence-making relations, one might fairly wonder what it is about the plane case that she thinks indicates any kind of understanding at all. As it happens, her view seems to be that no kind of understanding is interestingly different from what she sees as its corresponding type of knowledge, though her reasons are not immediately relevant for our current purposes.

understanding and/or a grasp of coherence-making relations). However, I think the case is nonetheless instructive insofar as it can tell us more about what is going on when 'understand that' is used instead of 'know that.'

Specifically, I think that these sorts of cases are simply instances of agents speaking somewhat lazily. While using 'understand' in this way usually allows speakers to roughly convey what they want to, this usage is not getting at the concept that epistemologists working on understanding are really interested in. Kvanvig makes a similar claim about what appear to be *non-factive* attributions of propositional knowledge – he thinks that such uses involve misspeaking, but concedes that if such uses become common enough then they will no longer be instances of misspeaking. However, by the same token so too will they have ceased to express anything about the concept of knowledge, and the word 'knows' will have come to express a different concept. I hold that what appear to be attributions of propositional understanding can be explained in a similar way. In the above case involving the flight cancellation, for example, the speaker is using 'understand' to refer to the concept of propositional knowledge (and we will shortly see speakers using 'understand that' to refer to other epistemic states). It seems that in Brogaard's specific example, the utterer should probably have said "I know that my flight was cancelled" in order to express the intended thought.[21] Consider that if a fellow traveller were to ask something like "Wait, don't you mean 'know'? If you understand something here, you must have more information about the flight cancellation than we do!" then it is likely that the agent would retract and correct his statement to reflect simply *knowing that* the flight had been cancelled.

The same sort of explanation applies (in the vast majority of cases) to statements such as "I understand that you are the person I should speak to about setting up an appointment" and "I understand that you need me to pick up some milk on the way home." This imprecise use of 'understand' to mean 'know' often works just fine in everyday conversation, but should not be taken to be importantly informative about the epistemic state of understanding with which epistemologists are concerned. After all, the nature and value of the ascribed epistemic state in such cases should surely be identical to that of propositional knowledge.

[21] I say this with the caveat that this is my contention about the plane case *unless* the speaker is trying to express his comprehension of *why* his flight was cancelled, in which case Brogaard's example becomes an instance of what we will see in a section five – a case of apparent propositional understanding in which what is meant is an attribution of *atomistic or objectual understanding*.

4. Hedging

Now, some cases featuring sentences of the form "I understand that X" do not seem at all explicable in terms of linguistically lazy references to propositional knowledge. For example, it seems as though Joan can say to her sister "I understand that the train leaves at seven o'clock" while both readily lacking knowledge and being aware that she lacks knowledge. So, since what is being attributed here is clearly not propositional knowledge, is the state referenced in such sentences what we should properly think of as genuine propositional understanding?

I think that these sorts of cases can be also be explained in such a way as to make it obvious that they do not feature genuine understanding (albeit for different reasons than those highlighted in the previous section). Specifically, I think what is present in the above example (and those like it) is not understanding but rather hedging for reasons of doubt. Since we have stipulated that in the train example Joan is aware that she lacks knowledge, I think it is plausible that she is using "I understand that the train leaves at seven o'clock" to convey something closer to "I think that the train probably leaves at seven o'clock but I have at least some cause to be hesitant about whether I am correct to think this." If pressed, it is seems likely that she would further explain her thoughts by revealing her doubt about the train times (explaining, perhaps, that she has not checked the most recent timetable changes, or offering other grounds that would make sense of her hesitancy to claim to have knowledge). Assuming again that the relevant agent does not (and would not claim to) have knowledge, the same sort of explanation can be given for an utterance like "I understand that the party will be a relatively small affair" in response to an anxious question about whether a party will be intimidating in its largeness, and a person's saying "I understand that they don't ask for ID" to her underage friend who wonders whether he will be allowed to enter a particular bar. The former agent, if pressed, would say something like "I have only heard of a few people saying that they have been invited, but I can't be sure," while the latter might offer something along the lines of "I was never asked for ID when I was your age, but the policies may be more strict now."

However, consider the following case that doesn't quite fit into the same category: Carl comes home much later than he previously claimed would be the case, and he asks his partner what is bothering her. She reproachfully replies "Well, I understand that we have an agreement about calling each other at times when one of us will not be coming home until after 1am." In such examples, it is not the case that the speaker feels unsure about whether she is correct – indeed, we can stipulate that she strongly remembers striking such an agreement and is

unhappy that Carl failed to hold up his end of their deal. This means that this is not an instance of hedging for reasons of doubt. Further, while the speaker may well have propositional knowledge and may well (in different circumstances) be willing to claim to have this knowledge, she is not here lazily using 'understand' to mean 'know.' Perhaps, then, this could be a case of legitimate, philosophically interesting propositional understanding?

Instead, I think that although these sorts of utterances do not feature hedging for reasons of doubt, they nonetheless involve hedging of a different kind. They are intended to at least slightly soften the potentially confrontational claim that one party's behaviour did not meet the other party's expectations (perhaps, in this specific case, out of a desire to be air a grievance and receive an apology without starting a fight).[22] If the speaker were to have her claim to understand questioned, she would be likely to rephrase her claim in some way that reflects that her choice of words is less about having *understanding* and more about her intention to communicate justified unhappiness without baldly accusing Carl of wrongdoing.[23] This second type of hedging relates to social conventions and the successful navigation of interpersonal relationships (and will therefore occur not just with romantic partners but also in the workplace, with family, with strangers, and so on). Another example might be of an employee politely prompting her boss by saying "I understand that we have established that my hours are to be reduced," or of a son saying to his mother "I understand that you are willing to end my being grounded now that I have done all of the household chores for a week."[24]

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[22] I base these examples on very roughly similar cases from Kvanvig, *The Value of Knowledge*, 191, who uses such cases for the alternative purpose of explaining away apparently *non-factive* uses of understanding. Grimm, "Understanding," 3, also agrees that there are cases where 'understanding that X' is used to suggest 'an openness to correction' (which fits with my first proposed kind of hedging, i.e. hedging for reasons of doubt).

[23] The second most likely answer that such a speaker would give would be to (instead or in addition) attribute to themselves some form of *atomistic* understanding (such as, in the Carl case, understanding *why* they struck their agreement about calling) or maybe even objectual understanding (such as understanding the rules of their romantic relationship). This alternative sort of retraction of claims to have propositional understanding will be discussed in the next section.

[24] Interestingly, there seems to be another way in which specifically *past tense* claims about propositional understanding can feature something in the neighbourhood of hedging. Here, I am thinking about self-exculpating and face-saving statements like "I understood that such activities weren't against the law" or "I understood that she wasn't seeing anyone at the time I asked her on a date." Such speakers are usually trying to defend their having had a belief that

5. Atomistic or Objectual Understanding

Finally, we should turn to a group of cases that feature agents of whom it is substantially more plausible to say that they have understanding. Consider, for example, an expert in scientific lab work. It does not at first seem at all inappropriate to say of him something like "He understands that he must be careful around these chemicals" without hedging in any way and also without intending to attribute mere propositional knowledge. Similarly, if Clint is very close to his wife of ten years it seems as though we might in some cases fairly say "Clint understands that Anna is happier now that she has a new job" without any intention to hedge and yet still be attributing something more substantial than propositional knowledge.

I think that one key to seeing the way in which these sorts of examples are not really representative of some distinctive epistemic state properly called propositional understanding is to focus on the extent to which the person handling chemicals and the thoughtful husband would not sum up their understanding with reference to just one proposition. Specifically, I think that the speakers in these cases (and cases like them) really mean to attribute *atomistic understanding* or *objectual understanding*. So, take an example of what might appear to be propositional understanding, such as "She understands that Gore might have been president." If the case involves an associated grasping of the coherence-making relationships relevant to *why* it is the case that Gore might have been president, it turns out to really feature atomistic understanding of why it is possible that Gore might have been president, and perhaps additional objectual understanding of the subject matter constituted by the relevant presidential election (if the subject of the sentence grasps enough of the coherence-making relationships relevant to the election). This means that when we say that the man handling particular chemicals understands that he must be careful around those chemicals, we could well be attributing to him atomistic understanding of why he needs to be careful around the chemicals, or objectual understanding of handling dangerous chemicals more generally (or even some other, larger subject matter that is relevant, such as chemistry). Similarly, when someone says that Clint understands that Anna is happier now that she has a new job, it is likely that what is meant is really that Clint has the awareness of his wife's psychology required to have understanding of *why* she is happier now that she has his new job, or that Clint understands something like the subject matter of

turned out to be false, suggesting that they had a highly justified false belief and are not to blame for not having known the truth.

his wife's career aspirations. Note once again that it seems plausible that the speakers would rephrase their claims in ways similar to those I have just described here if pressed on the matter of exactly *what* they understand. They would be unlikely to simply staunchly insist "That Anna is happier now!" or "That he must be careful around these chemicals!"

6. Concluding Remarks

It is easy to make the unhelpful assumption that each type of understanding stands in contrast with a corresponding type of knowledge. By looking specifically at what propositional understanding might be, we have seen that the idea that there is such understanding (in an epistemologically interesting sense) is implausible. Firstly, the most sensible picture of what propositional understanding might be does not clearly describe a type of understand that actually takes a proposition as its object. Secondly, attributions of propositional understanding are largely (i) synonymous with attributions of propositional knowledge, (ii) cases of hedging for reasons of doubt, (iii) cases of hedging for reasons of social convention, (iv) really attributions of atomistic understanding, or (v) really attributions of objectual understanding (where (iv) and (v) both involve more than just one proposition and also seem to involve grasping coherence-making relationships). Further, I would contend that any other types of cases where propositional understanding is attributed will also be cases in which a widespread willingness to retract and rephrase such claims will show that something other than propositional understanding is what is really being attributed.

At this point, driven by the conviction that there is no such thing as genuine instances of propositional understanding, I suggest that we abandon the idea that this is an important breed of understanding that warrants further, in-depth consideration as part of the project of investigating the nature of understanding. Without the unhelpful and confusing notion of propositional understanding in play, we will be much better placed to make real progress in discovering what is distinctive about the cognitive achievement of understanding, and this in turn will help us to learn more about why it might be particularly valuable.

THE LOGICAL LIMITS OF SCIENTIFIC KNOWLEDGE: HISTORICAL AND INTEGRATIVE PERSPECTIVES

Ettore DE MONTE, Antonino TAMBURELLO

ABSTRACT: This work investigates some of the most important logical limits of scientific knowledge. We argue that scientific knowledge is based on different logical forms and paradigms. The logical forms, which represent the rational structure of scientific knowledge, show their limits through logical antinomies. The paradigms, which represent the scientific points of view on the world, show their limits through the theoretical anomalies. When these limits arise in science and when scientists become fully and deeply aware of them, they can determine logical or paradigmatic revolutions. These are different in their respective courses, although the logical forms and the paradigms are parts of the same type of knowledge. In the end, science can avoid or can integrate its different limits. In fact, the limits of science can become new opportunities for its growth and development.

KEYWORDS: limits, scientific knowledge, paradigm, logical form, scientific revolution

1. Introduction

In this work, we will investigate the limits of scientific knowledge and in particular its logical limits. We believe, in fact, that scientific knowledge can be described as being composed of a basilar logical form and of one or few paradigms (in the classical, kuhnian, sense of this term). While the logical form refers to the elaboration of knowledge (rational style and structure), the paradigm refers instead to the representation of the same knowledge (point of view on the world, theories and methodologies). Scientific knowledge is limited in both its elaborative and representative capacities. In other words, it shows limits both in its logical forms (logical limits) and in its paradigms (paradigmatic limits). We understand the concept of 'limit' in terms of a border, or a barrier that delimits the extension of a specific field of knowledge.

A large part of our study consists in discussions of the most important and most recent philosophical works on its topic. We will discuss in particular the limits of knowledge listed and described by the Princeton Group (Piet Hut, David

P. Ruelle, and Joseph F. Traub) and the limits of thought listed and described by Graham Priest within the new paraconsistent logic. We believe that the first type of these limits can be interpreted as elaboration limits of scientific knowledge, and so they can be described as logical limits. Moreover, we believe that the second type of these limits can be interpreted as limits of representation of scientific knowledge, and so they can be intended as paradigmatic limits. From our point of view, however, logical and paradigmatic limits cannot be understood in disjoint terms, although they can be studied separately. Therefore, evidently, each limit can have a double interpretation: 1) a specific reading that distinguishes the logical limits and the paradigmatic limits of scientific knowledge; 2) a general reading that considers the limits of scientific knowledge as a whole with elaborative and representative features. In any case, the concept of limit should not suggest the idea of something insurmountable. In fact, the logical and paradigmatic limits of scientific knowledge determine the field of extension of the acquiring capacities. The acquisition of new knowledge is limited by certain styles of rational elaboration and by certain patterns of world representation. What is beyond these limits is not covered and it cannot be acquired. But this condition marks the possible transition of science to phases of change or to real revolutions, as evidenced by Kuhn. In fact, the new discoveries and the new acquisitions can cause the breaking and the overtaking of the dominant limits, so developing new logical forms and new paradigms that inevitably will develop new limits. Hence, we can understand the nature of the limits of scientific knowledge as being, at the same time, both rigid and elastic.

In what follows, we will focus our efforts especially on logical forms, on logical limits and their foundations. The arguments related to these matters are, in fact, relatively new, and so we believe that their deeper exposure is at least useful, if not necessary.

2. The Logical Forms of Scientific Knowledge

From a historical point of view, scientific knowledge takes different logical forms. We can operationally define the logical form in the terms of a structure, a configuration, or a rational style that scientific knowledge can take in its history.

1) We call the first logical form *Strong Deductivism*. It is based on an axiomatic logic inspired by an idealistic philosophy. It is typical of the more systematic tendencies of science that plunge into Euclidean Geometry and take expression in modern formalist approaches. The theoretical model of strong, or exclusive, deduction implies the massive intervention of the operative and instrumental mental mechanisms. Deduction, lacking of investigative qualities, is a simple

application to reality of the aprioristic and pre-existing certainties that are irrelevant to experiences and to learning. The mind explains the reality in a rational way, by identifying some of its well-established truths. This theoretical setting translates deduction into a recognition mechanism of the world. It is exclusive because it excludes all the mental processes aimed at discoveries. It is strong because, from its point of view, deduction has no need of anything more than itself to be complete.

2) We call the second logical form *Weak Deductivism*. It is based on a formal logic inspired by an empiricist philosophy. This is typical of the positivistic and neo-positivistic tendencies of science that are impressed by the discoveries of contemporary quantum physics. The theoretical model of weak, or inclusive, deduction lightens the burden of innate ideas. It refers the certainty of knowledge to a hard, complex and demanding act of study, ideation and investigation of the reality. So, knowledge evolves gradually: through the ages, it avails itself of amazing works of genial personalities able to enlarge the mass of knowledge through their intuitions and clarifications. Deduction is a part of the largest process of rationality and a conclusive phase aimed to tidy up all knowledge, already learned through a hard inductive work of investigation starting from available data in reality. Deduction has the characteristics of a knowledge accommodation mechanism. It is strongly inclusive because it needs the intervention of mental processes aimed at axioms, discoveries, and creations. It is weak because, from its point of view, deduction needs something more than itself to be complete.
3) We call the third logical form *Pure Inductivism*. It is based on a hypothetical logic inspired by an infinitary philosophy. This feature is typical of all the relativistic tendencies of science that reject the absolute value of knowledge. In fact, knowledge has no boundaries and no limits. No mechanism is able to stem its endless flow. This continuity is unstoppable and not bound in a delimited and discrete field of knowledge. Calculability loses its value and at the same time the infinite dilution of all meanings dominates all. Induction flows pure, without obstacles. Inductivism sets the idea of inexhaustible empiricism as its implicit axiomatic base. In this logical form, empiricism is an axiom of inexhaustibility.

Each of these trends implies some logical limits that are evident in paradoxes and that arise from their own philosophical basis.

3. Strong Deductivism

Axiomatic logic is typically deductive. It is structured around theoretical certainties and it uses closed methodologies. Reasoning aspires to the perfection of its knowledge systems, avoiding an infinity reference of demonstrations and theorizations.

Each perfect theory explicates irrefutable truths in following these criteria:

- Argumentative expressivity that shows all the contents.
- Decidability of method that verifies or confutes all the statements using algorithmic modalities.
- Coherence of reasoning that emerges from brief statements without a contradiction.
- Completeness of judgements that includes all the possible truths of a particular argument.

David Hilbert is the greatest exponent of this idea in 20th century. Thanks to him, a new logical tendency rises: Formalism. He proposes the idea of closed theoretical systems structured around a finite numbers of symbols and symbol relations. These systems are able to represent knowledge in an exhaustive way.

In 1900, during the "Second International Congress of Mathematicians in Paris," Hilbert presents a famous dissertation about contemporary history of mathematics. He proposes a list of 23 problematic questions. The second of these questions asks to demonstrate the coherence of arithmetic. Hilbert wants to avoid the risk of having logical contradictions. Coherence becomes a necessary parameter of demonstrative rigor. Few years later, in 1928, during the "International Congress of Mathematics in Bologna," Hilbert[1] takes on the ambitious task of giving to mathematics a full and final formal shape, in fixing it as a homogeneous and structured system. He thinks to a mechanical procedure able to resolve all mathematical problems and that belongs to a defined symbolic class.

Hilbert[2] challenges the mathematical academic world, starting from his concerns about non-Euclidean[3] geometries and paradoxes, factors of an undoubted logical weakness. So, he believes that is necessary a complete formalization of the whole discipline. In this way, mathematics would be reduced to a pure system of

[1] David Hilbert, "Problemi della fondazione della matematica," in *Ricerche sui fondamenti della matematica*, ed. V. Michele Abrusci (Naples: Bibliopolis, 1978), 291-300.

[2] David Hilbert, "Nuova fondazione della matematica. Prima comunicazione," in *Ricerche sui fondamenti della matematica*, 189-213.

[3] The non-Euclidean geometries arise in reaction to the Euclid's fifth postulate, the only one more intuitive (Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (New York: Dover, 1956)).

axioms, symbols, formulas, rules and demonstrations. In fact, for Hilbert, the paradox found in some logical formulations is due to their semantic value. Logic and mathematics are 'polluted' by the reference to meaning of words and of represented things. Therefore, he believes that is necessary to create a meaningless frame. So any statement about a system becomes the product of meta-mathematical observations performed in according to a language that is outside of the mathematical language itself. So, we can advance statements about the characteristics of the mathematical system with the use of the terms and the rules of another language. For example, if we admit that the formula "*x+0=0*" is a theorem of the formal arithmetical system, then we are not talking about the mathematical semantics, but we are talking about a syntactic characteristic of mathematics in the terms of a language (english) that is different from the mathematical one. For Hilbert,[4] meta-mathematics is outlined like a new branch of mathematics, interested in what it is possible, or not, to demonstrate.

According to Hilbert, a formal mathematics respects coherence, completeness, expressivity and decidability. In this way, the non-Euclidean presuppositions would collapse and the risk of a not delimited infinity would be avoided. Hilbert's model is, in fact, finite because it loses the inconvenient concept of infinity in act.[5] It follows the regular flow of its demonstrative methods.[6] Mathematics must not refer to the actually infinity objects, but only to the potentially infinite collections as the natural number sequence (*N*). The problem of foundations[7] originates when reason goes beyond abstraction and idealization limits, arriving to the transcendental notions that lead to paradoxes and not empirical geometries. The idea of the infinity is uncertain, fleeting and not theorizing. When the semantic infinite drift is established[8] and meta-mathematics is limited to a syntactic level, mathematics would be anchored to a formal and symbolic finitude. So, a

[4] David Hilbert, "I fondamenti logici della matematica," in *Ricerche sui fondamenti della matematica*, 215-31.

[5] The distinction between the actual infinity and the potential infinity dates back to Aristotle. The first term indicates that the infinity is something active. The second term considers the infinity as a purely cognitive instrument (Jeanne-Pierre Luminet, Marc Lachiéze-Rey, *De l'infini… Mystéres el limites de l'Univers* (Paris: Dunod, 2005)).

[6] David Hilbert, "Sull'infinito," in *Ricerche sui fondamenti della matematica*, 233-66, "Conoscenza della natura e logica," in *Ricerche sui fondamenti della matematica*, 301-11.

[7] The dispute around the mathematical foundations arises between the 19th and the 20th centuries.

[8] David Hilbert, "La fondazione della Teoria Elementare dei Numeri," in *Ricerche sui fondamenti della matematica*, 313-23.

mathematician must respect the *Finiteness Theorem* that ensures the exclusive derivation of the theoretical consequences from some finite axiomatic premises.

For Hilbert,[9] axiomatic logic is an unrepeatable chance for the growth of science. It facilitates the overtaking of all the epistemological doubts around foundations in referring to sign and formalization.[10] Moreover, it prevents the contradictory that is, instead, typical of immature sciences.[11] So, each symbolic and axiomatic system represents a closed logic.

This sort of logical closure brings some undoubted advantages: 1) Simplicity. The limited vocabulary of terms is a good instrument and as a linear method is a good procedure;[12] 2) Conceptual representation. Each idea is an evident certainty. It takes concreteness and it becomes a clearly element even if only symbolic;[13] 3) Essentiality. All ideas and statements of a formalized system are logical nuclei that create a complicate and interactive matrix.[14] But, beyond the advantages, closed logic leads to a no good knowledge fragmentation. Formalism produces, in fact, several logical systems that respond to perfect prerequisites.

3.1 The Idealistic Limits of Strong Deductivism and the Open Logic

Kuert Gödel[15] proves that arithmetic cannot be completely formalized. The *Arithmetical Incompleteness Theorems* establish that any closed formal system, independently from its breadth, is necessarily incomplete, because any statement concerning its internal character cannot come from its own language, but from another one, outside of it. Gödel's discovery shows how any mathematical system, even if apparently complete and correct, includes some statements that are not

[9] David Hilbert, "Pensiero assiomatico," in *Ricerche sui fondamenti della matematica*, 177-88.

[10] David Hilbert, "Sui fondamenti della logica e dell'aritmetica," in *Ricerche sui fondamenti della matematica*, 163-75, "Nuova fondazione della matematica," "I fondamenti della matematica," in *Ricerche sui fondamenti della matematica*, 267-89.

[11] David Hilbert, "Problemi matematici," in *Ricerche sui fondamenti della matematica*, 145-62, "Pensiero assiomatico," "Nuova fondazione della matematica," "Dimostrazione del Tertium non Datur," in *Ricerche sui fondamenti della matematica*, 325-30

[12] Hilbert, "Problemi matematici."

[13] Hilbert, "Pensiero assiomatico."

[14] Hilbert, "Pensiero assiomatico," "Conoscenza della natura," David Hilbert and Paul Bernays, "Grundlagen der Mathematik," in *Ricerche sui fondamenti della matematica*, 329-474.

[15] Kurt Gödel, "On Formally Undecidable Propositions of Principia Mathematica and Related Systems I," in *Kurt Gödel. Collected Works Volume I: Publications 1929-1936*, eds. Solomon Feferman, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort (New York: Oxford University Press, 1968), 144-95. The first public formulation of Gödel's theorems dates back to October 7, 1930, during a conference in Königsberg.

demonstrable or refutable by its own logical tools. The truth of these statements is not resolvable through the procedures allowed by the system. Gödel discovers that deductive logic does not consent to prove all the possible true relationships between the numbers. The truth always goes beyond demonstrations. Some propositions cannot be proven true. Formalism is thus destroyed.

In the attempt to resolve the second great question of Hilbert's list, Gödel[16] studies arithmetical coherence and the problems that it entails. He is soon faced with the extreme certainties of Formalism. Anti-coherence, or paradox, is a special self-referral idea. When a system states something about itself, it produces irresolvable statements, on which it is not possible to demonstrate truth or falsity. Thanks to Gödel, mathematics talks about itself and so it let itself to fall into a logical self-reference.

A formalized mathematical system is, at the same time, a set of uninterpreted symbols and a set of linguistically interpreted symbols. In accord with Hilbert, Gödel distinguishes the object-language of numbers and the meta-language of the ordinary words that describes the same numbers. But, differently from Hilbert, he links theory and meta-theory. So, if numbers are, at the same time, terms of object-language and meta-language, then mathematics talks about itself and it becomes a self-referral system.[17] Gödelization, or Gödel's numbering,[18] is the procedure that allows the mathematical self-reference through the arithmetization of the meta-theory. It is an ingenious method that transforms logical variables into numerical variables.

In *Principia Mathematica* of 1910-1913, Alfred North Whitehead and Bertrand Russell[19] create a complex logical system that represents arithmetic as a good meta-language. In this model, the symbols express syntax, in talking about numbers and their relationships.[20] Gödel translates this logical system into an arithmetical system. Each of its logical entities is transformed, in fact, into a number, so at the end numbers will talk about other numbers. Formulas and logical demonstrations are associated with particular arithmetical notions (Gödel's

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[16] Kurt Gödel, "On the Completeness of the Calculus of Logic," "The Completeness of the Axioms of the Functional Calculus of Logic," "On the Completeness of the Calculus of Logic," "Some Metamathematical Results on Completeness and Consistency," in *Kurt Gödel. Collected Works Volume I*, 61-101, 102-23, 124-5, 140-3.

[17] Gödel, "On Formally Undecidable Propositions of Principia Mathematica."

[18] The technique is also known as Diagonalization, Syntax Arithmetization or Fixed Point.

[19] Alfred N. Whitehead and Bertrand A. W. Russell, *Principia Mathematica* (Cambridge: Cambridge University Press, 1962).

[20] Gödel, "The Completeness of the Axioms," "On Formally Undecidable Propositions of Principia Mathematica," "On the Completeness."

numbers). This method gives a natural number to each discrete object of a logical system. Any symbol (ε), formula or string receives its Gödel's number, or Gödelian $g(\varepsilon)$. Gödelization gives various numbers to various expressions, so it is also an algorithmic procedure. If we have any expression (ε), then we can immediately calculate its number $g(\varepsilon)$, and if we have any natural number (n), then we can immediately identify an expression (ε) for which $n=g(\varepsilon)$.

Thanks to this self-reference, Gödel discovers the logical paradox of any system based on numbers. He translates Epimenides' revised form of the classical Eubulides' paradox, "This sentence is false" in the assertion "This sentence is not demonstrable." So, he translates this new assertion in numerical terms (Gödelian statement or *G*). If it is demonstrable then it is true, but based on what it says, it is not demonstrable. On the contrary, if it is not demonstrable then it is false, but contradicting what itself says, it is demonstrable. Thus, mathematics is opened to logical paradoxes and it cannot be entirely coherent and complete.[21]

So Gödel formulates his two famous theorems that, after some years,[22] he will describe in informal way as his most important logical discoveries. On the basis of the *First Incompleteness Theorem* (*G1*), we can affirm that, in the language (*L*) of a correct mathematical system (*S*), an undecidable statement (*G*) inevitably arises. *G* is not demonstrable or rebuttable within *S* and with the language of *S* (*L*). The completeness of a mathematical system is not achievable. Instead, on the basis of the *Second Incompleteness Theorem* (*G2*) that derives from the first, we can affirm that if *S* is a correct mathematical system, then *S* will not prove its internal coherence. This theorem causes the final collapse of any formalist illusion and it gives a negative answer to the question on coherence presented by Hilbert in Paris. We cannot prove the coherence of any logical system if we are into the same system.

An interesting generalization of Gödel's discoveries comes from the developments of logic and from his same statements.[23] During thirties, Alan

[21] Gödel, "On Formally Undecidable Propositions of Principia Mathematica," "Lecture on Completeness of the Functional Calculus," in *Kurt Gödel. Collected Works Volume III: Unpublished Essays and Lectures*, eds. Solomon Feferman, John W. Dawson Jr., Warren D. Goldfarb, Charles D. Parsons, and Robert M. Solovay (New York: Oxford University Press, 1995), 16-29, "On Undecidable Sentences," in *Kurt Gödel. Collected Works Volume III*, 30-5.

[22] Kurt Gödel, "Some Basic Theorems on the Foundations of Mathematics and their Implications," in *Kurt Gödel. Collected Works Volume III*, 304-23.

[23] Kurt Gödel, "Postscript to Spector 1962," in *Kurt Gödel. Collected Works Volume II: Publications 1938-1974*, eds. Solomon Feferman, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort (New York: Oxford

Turing's works help and support Gödel's Theorems. Turing proposes a good solution to the definition of calculability, so reinforcing Godel's *Incompleteness Theorems*. In fact, decidability, which is the base of these theorems, is associated with the idea of calculability. So, a lack of definition of this idea is a weakness. Turing[24] idealizes a calculating machine and he defines "calculation" all the process that this machine makes. Calculation is intended as the ability to keep in mind a set of rules. It is an algorithmic, mechanical and linear procedure. Turing's Machine (TM) is characterized by finite sets of states and instructions that move a reading and writing head along the compartments of a potentially infinite tape. Gödel thinks that the TM is a rigorous and adequate demonstration of the notion of effective procedure. Above all, thanks to this mechanical model, the idea of incompleteness regards each formal system defined by the mechanical production of theorems. A formal system is constituted by rules that transform some statements in other statements and that can be directly applied by a human agent or indirectly incorporated and executed by a machine. The necessity of arithmetization disappears and the phenomenon of incompleteness seems really belong to each mechanized method. Gödel[25] admits that his theorems are applicable to all formal-logic systems and not only to mathematical ones. Each logical system must be syntactically and semantically opened.

Gödel[26] demolishes the strong idea of Formalism and develops a sort of open logic that surpasses the idea that man can know irrefutable scientific truths. Truth is only partial and perfectible. For Gödel,[27] a formal system cannot be closed and it cannot entrench itself in certainties that are held as always valid. The universe of ideas transcends human knowledge.

The idealistic limitation that Gödel imposes to Formalism is, nearly, confirmed by the results of other important logicians. Alfred Tarski[28] defines the

University Press, 1990), 253, "What is Cantor's Continuum Problem?" in *Kurt Gödel. Collected Works Volume II*, 254-70.

[24] Alan M. Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society* s2-42, 1 (1937): 230-65, "On Computable Numbers, with an Application to the Entscheidungsproblem. A correction." *Proceedings of the London Mathematical Society* s2-43, 1 (1938): 544-6.

[25] Gödel, "Postscript to Spector," What is Cantor's Continuum Problem?"

[26] Gödel, "Some Basic Theorems."

[27] Gödel, "On a Hitherto Unutilized Extension of the Finitary Standpoint," in *Kurt Gödel. Collected Works Volume II*, 240-1, "On an Extension of Finitary Mathematics which has not yet been Used," in *Kurt Gödel. Collected Works Volume II*, 271-80.

[28] Alfred Tarski, *Pojecie Prawdy w Jezykach Nauk Dedukcyjnych* (Warszawa: Nakladem Towarzystwa Naukowego Warszawskiego, 1933), "The Concept of Truth in Formalized

syntactic and semantic opening of a formal system in terms of the linguistic higher order evaluation. Tarski creates the *Undefinability Theorem* according to which arithmetic cannot express and define some numbers properties, as the concept of truth. So, arithmetic loses some contents and the formalist assumption of argumentative expressivity. Finally, Alonzo Church[29] and Barkley Rosser[30] create the *Undecidability Theorem*. It establishes that, starting from some theoretical axioms, we cannot create a general algorithm that is able to verify, or not, each specific logical content. With the further collapse of logical decidability myth, Hilbert's proposal to axiomatize mathematics and science in general is not realizable.

Gödel's results reveal that the limits of axiomatic models come from the excesses of rigor. These are signs of reductive mentalities conditioned by the myth of syntax and the underestimation of semantic.[31] The man cannot create correct and complete theories of universe.[32] Science is the final judge of truth, but it must also recognize and elaborate its limitations.[33]

3.2 The Foundations of Open Logic

Beyond non-Euclidean geometry, a revolutionary moment in the history of logic is the discovery of Russell's paradoxes that confute the set theory of Gottlob Frege. In a first moment, Frege and Russell refuse Giuseppe Peano's theoretical system[34] because of its fragility, its unclarity in demonstrative steps and real priority of its

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Languages," in *Logic, Semantics, Metamathematics*, ed. John Corcoran (Indianapolis: Hackett, 1983), 152-278.

[29] Alonzo Church, "An Unsolvable Problem of Elementary Number Theory," *American Journal of Mathematics* 58, 2 (1936): 345-363, *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956).

[30] Barkley J. Rosser, "Extension of Some Theorems of Gödel and Church," *Journal of Symbolic Logic* 1, 3 (1936): 87-91, "An Informal Exposition of Proofs of Gödel's Theorem and Church's Theorem," *Journal of Symbolic Logic* 4, 2 (1939): 53-60.

[31] R. Rosen, "Undecidability and Biology," in *On Limits*, eds. John L. Casti and Joseph F. Traub (Santa Fe Institute: Teleconferences (Report 94-10-056), 1994), 15-6.

[32] Stephen Hetherington, "Knowledge's Boundary Problem," *Synthese* 150, 1 (2006): 41-56.

[33] Christopher Cherniak, "Limits for Knowledge." *Philosophical Studies* 49, 1 (1986): 1-18.

[34] In 1889, Peano (Giuseppe Peano, "The Principles of Arithmetic, Presented by a New Method," in *A Source Book in Mathematical Logic*, ed. Jean van Heijenoort (Harvard: Harvard University Press, 1967), 83-97) tries to axiomatize the arithmetic, in retracing the Euclidean geometrical model. Primitive concepts: zero; number; successor. Axioms: *1* is a number; the successor of a number is another number; if two numbers are equal, then they have the same successor; *1* is not the successor of another number; if *k* is a set, *1* is part of *k* and, for each number (*x*), if *x* is part of *k*, and also *x+1* is part of *k*, then *k* contains the entire class of *N*.

basilar concepts. So, these two authors inaugurate the logicist program proposing to bring mathematics to its logical and primary bases. This is also a rigid axiomatization that is, however, focused on the aprioristic set theory. Frege and Russell sustain that sets are more primitive concepts than numbers. In fact, numbers are always definable in terms of sets. Each number can be reduced to a most primordial notion that is only guess. The *Theory of Complex Sets*, created by Frege,[35] postulates that each mathematical element can be included in a specific set that is, also, a part of a wider set. This is a closed structure where we can insert any mathematical statement. There are sets that refer to other sets and that close everything in general axiomatic wholes that are, in turn, confined into more extensive sets. At some point, concatenation should end with the absolute wider set that can include each other set, even itself. But this epilogue represents the maximum limitation of this closed logical formulation.

The same Russell[36] discovers a deep antinomy inside this set theory. R is the set of all sets not members of themselves. R is a collection of sets, so an x-set can be part of it if and only if it is not a member of itself. But is R a member of itself, or not? If it is not a member of itself, then it is part of R, because R is constituted, to definition, from all sets not members of themselves. In this case, R is part of itself, but this affirmation contradicts the early statement. Instead, if R is a member of itself, then it is a part of sets not members of themselves; R is not a part of itself.[37]

In *Principia Mathematica*, Russell tries to save the Logicism from paradoxes, elaborating the *Logical Types Theory*, which Gödel[38] well describes. Russell argues that each set is part of a level that is higher than the level that includes its same constitutive elements. So, the concept of a set that includes all sets is a mistake. Russell renounces to the idea of sets for the new idea of types that assumes two different versions: a simple one and a complex, or branched, one. Paradoxes arise from a vicious circle consisting in the supposition that a collection of objects can contain members whose definitions derive from the collection intended as a whole. To avoid this logical self-reference of totality, or set, to itself we must formulate some statements not included inside its range of references.[39] The paradox of sets arises from the belief that all sets are part of the same type.

[35] Gottlob F. L. Frege, *The Basic Laws of Arithmetic* (Berkeley: University of California Press, 1964).

[36] Bertrand A.W. Russell, "Letter to Frege," in *From Frege to Gödel*, ed. Jean Van Heijenoort (Cambridge: Harvard University Press, 1967), 124-5.

[37] X is a class. R is a set of collection defined as $(X \in R) \leftrightarrow (X \notin X)$. But $X=R$, because R is however a class. So, R is defined as $(R \in R) \leftrightarrow (R \notin R)$.

[38] Kurt Gödel, "Russell's Mathematical Logic," in *Kurt Gödel. Collected Works Volume II*, 119-43.

[39] Bertrand A.W. Russell, *The Philosophy of Logical Atomism* (Peru: Open Court, 1985).

Whereas, we must understand that the properties of a higher type are applied, or predicate, only to the objects of a lower type.[40]

Thanks to Russell and Gödel's works, we can know the inexhaustibility character of open logic. In fact, if we decide to add the Gödelian undecidable statement (G) among the axiomatic system, so creating a more powerful system, however the new system will have its own undecidable statement. So, we can consider the formula *"$S=\gamma$"*, where S is a system identified with its undecidable statement (γ). Then, we can consider γ as an axiom of another powerful system (S_1). But also in this case, the presence of another undecidable statement (γ_1) would be proved: $S_1=(S+\gamma)+\gamma_1$. We can proceed further and the results would delineate a regularity such as $S_2=(S_1+\gamma_1)+\gamma_2$, $S_3=(S_2+\gamma_2)+\gamma_3$ $S_4=(S_3+\gamma_3)+\gamma_4$….., $S_n=(S_{(n-1)}+\gamma_{(n-1)})+\gamma_n$. At a certain point, the mind cannot go beyond. If a formal system indefinitely postpones its undecidability, then the human mind could not see its coherence. Everything would turn into its origin ($S_\infty=\gamma_\infty$) and confirms the undecidability of the infinity.[41]

The logical infinity is an essential aspect of human knowledge. It doesn't set limits, but it exalts the fallibility of each certainty that is limited to closed fields. Infinity delays to not delineable horizons and causes the end of all cognitive illusion of infallibility.

4. Weak Deductivism

The empirical perspective considers and preserves data without transcending their concrete level:

1) The objectivity of the observation aspires to capture data, without any subjective interference. The observer must dismiss his subjectivity, becoming a mechanical container of external occurrences.

2) The certainty of reason is structured on objective data. Thanks to an inductive procedure, the mind elaborates any fact and the totality of them to formalize some certain principles. These principles are similar to the idealistic axioms but different because they come from the experiences.

This empirical logic finds fruitful ground among the followers of neo-Positivism. The theorists of Vienna Circle are the most important exponents of this perspective. They were a group of scholars interested in an objective evaluation of reality and in abjure any speculative tendency. Philosophy must

[40] Russell, *The Philosophy of Logical Atomism*,

[41] Gödel. "Lecture on Completeness," "The Present Situation in the Foundations of Mathematics," in *Kurt Gödel. Collected Works Volume III*, 45-53

leave the worldviews elaboration to concentrate itself on a conceptual clarification. Real scientific success is inside the encounter between empirical observation, which constitutes the objective base, and the subsequent mathematical deduction.[42] In accord with Auguste Comte's positivism and with the empiricism of modern age, neopositivists think that knowledge must be based on scientific experience and it must be explicated thorough a symbolic logic.[43] In a formalistic way, science can take a precise and formal representation. Logical neo-positivism distinguishes scientific phase of discovery and scientific phase of justification. The first one is a non-logical elaboration of hypotheses, because discovery doesn't possess effective rules.[44] The second one, however, is a purely evaluation of the same hypotheses, because it merges data and theories.[45] Thus, some knowledge courses become indispensable: 1) signification of scientific terms; 2) nomological and deductive explication; 3) hypothetical and deductive justification; 4) theoretical axiomatization. The verification of meaning follows the perfection of terminology definition,[46] to avoid the risk of a conceptual confusion. The meaningless scientific assertions are not false, but they are incomprehensible. So, observational propositions become essential. Their value of truth derives from sensorial immediacy. These propositions represent the states of the physical world, in reducing observation to a realistic physicalism. The other propositions, which express some unobservable concepts (ex. physical strength), can be indirectly verified from the same observable propositions. The new knowledge must be reduced to the symbols used to represent the old knowledge.[47] Percy Bridgman's Operationism,[48] for example, leads us to consider each new scientific term contained into some propositions that can be confirmed or confuted thorough some operations. When the meaning of the term is established, we can apply deductive explication to formulate some schematic explications, typically mathematical and nomological.[49] These explications derive from the known statements that are both primary, because they express scientific laws, and secondary, because they describe empirical facts intended as conditions of knowledge. Laws are conditional assertions ("if x happens, then y happens"),

[42] Hans Reichenbach, *The Rise of Scientific Philosophy* (Berkeley: University of California Press, 1951).
[43] William Bechtel, *Philosophy of Science* (Nillsdale: Lawrence Erlbaum, 1988).
[44] Reichenbach, *The Rise of Scientific Philosophy*.
[45] Reichenbach, *The Rise of Scientific Philosophy*.
[46] Moritz Schlick, *Forma e contenuto* (Turin: Bollati Boringhieri, 2008).
[47] Schlick, *Forma e contenuto.*
[48] Percy W. Bridgman, *The Logic of Modern Physics* (New York: Beaufort, 1927).
[49] Schlick, *Forma e contenuto.*

which exclude any exception.[50] The initial conditions affirm the prior fact's occurrence ("*x* has happened"). So explication becomes, through *modus ponens*, a clear deductive conclusion (then *y* happens). Hypothetical and deductive method becomes necessary to identify scientific laws. Starting from an event that needs clarifying explications, scholars propose some verifiable hypotheses on the base of a continuous testing. Therefore, certain hypotheses, which are inductive and probabilistic processes of data enumeration,[51] are compared to scientific laws.[52] Finally, the strong axiomatic perspective of all the logical neo-Positivism takes evidence and it reduces empirical dimension to a closed system. In contrast to its same initial intentions, Neo-Positivism follows a Euclidean logic. In fact, when it affirms that a logical law is explicated, it reduces it to a reference theory. At the same way, Neo-Positivism explicates the events, in reducing them to a logical law.[53] This is a closed and circular course in which theories are deductive structures with primitive terms and these axioms assumes a logical laws form. Science must be divided in forms and structures, thus to be quantifiable.[54]

Empirical and axiomatic logic is objective and certain, limited and obstructed. Therefore it is a closed logic. This logic doesn't differ from deductive and exclusive logic, that is immolated to the myths of coherence and completeness. According to logical empiricism, induction is a rational searching of truths that uses intuition to recognize the theoretical primacy of each concept.[55] The true axioms are always evident, enveloped and limited.

Weak Deductivism associates axioms and experiences and so, in this perspective, rigidity has a different meaning. This logic doesn't neglect the possibility of changes, because it considers axiomatization an ultimate target of certainty and not something already known. This is a challenging course that consists of difficult confirmations, but also of many refutations. The meaning of *Weak Deductivism* emphasizes that a procedure cannot betray the value of each human knowledge experience. The idea of axiomatic perfection predisposes this logic to closure, losing the opportunity to consider its own opening.

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[50] Reichenbach, *The Rise of Scientific Philosophy*.

[51] Carl G. Hempel, "Studies in the Logic of Confirmation I," *Mind* 54, 213 (1945): 1-26, "Studies in the Logic of Confirmation II," *Mind* 54, 214 (1945): 97-121, *Aspects of Scientific Explanation and other Essays in the Philosophy of Science* (New York: Free Press, 1965), Reichenbach, *The Rise of Scientific Philosophy*, Schlick, *Forma e contenuto.*

[52] Schlick, *Forma e contenuto.*

[53] Bechtel, *Philosophy of Science.*

[54] Schlick, *Forma e contenuto.*

[55] Reichenbach, *The Rise of Scientific Philosophy*.

During 20th century, this logic has other estimators: 1) the Bourbaki Group[56] believes that any axiomatic system must be proved on the base of experience; 2) Haskell Curry,[57] a founding father of combinatory logic,[58] thinks that formal modality is the most important aim of any logical system. Each logical structure can be considered valid until it doesn't contrast with another data that activate the confutation process and a subsequent reformulation; 3) Saunders Mac Lane[59] considers each axiomatic system as an intuitive result of human activities that assumes theoretical consistency and, at the same time, it deviates away from these same activities. In these scholars, axiomatization encounters an empirical foundation and it acquires mutability and flexibility. However, it doesn't lose its essential narrowing quality.

4.1. The Empiricist Limits of Weak Deductivism and Hypothetical Logic

Inclusive approaches fall in a logical closure. In an attempt to contrast idealistic abstractionism, these approaches assume an empiricist perspective without detracting the domain of deductive method. In fact, when the axioms are objectively identified, the subsequent deductive rationalization remains intact. But, according to Jackson,[60] if two of the most important limits of knowledge arise from reason constraints and from relationship between the subjective world and the objective one, then inclusive empiricism is doubly limited. In fact, it follows both axiomatic perfection and empirical certainty.

The objectivity of observation is the first focal concept of this type of logic. The term 'objective' refers to what is concrete, because it has not subjective implication. An evident contradiction is already inherent: the exclusive relationship between observation and objectivity. Anyway, observation is always a human act, a subjectivity product. The same assertion can be valid for the observational instruments that can replace human acts, in introducing more assurances of validity, but that remain always human creations. The act of observation cannot be totally objective because it is, above all, a subjective act.

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[56] Nicolas Bourbaki, *Eléments de Mathématique* (Paris: Hermann, 1939).

[57] Haskell B. Curry, *Outlines of a Formalist Philosophy of Mathematics* (Amsterdam: North Holland, 1951), *Foundations of Mathematical Logic* (Mineola: Dover, 1979).

[58] Haskell B. Curry, Robert Feys, *Combinatory Logic I* (Amsterdam: North Holland, 1958), Haskell B. Curry, J. Roger Hindley, and Jonathan P. Seldin, *Combinatory Logic II* (Amsterdam: North Holland, 1972).

[59] Saunders Mac Lane, *Mathematics, Form and Function* (New York: Springer-Verlag, 1986).

[60] E. Attlee Jackson, "Final Comments in the Workshop Limits to Scientific Knowledge," in *On Limits*, 18-9.

An important confirmation to the paradox of the objectivity comes from the quantum physics of the 20th century. Werner Heisenberg[61] formulates the *Uncertainty Principle*, an important inducement for contemporary philosophical reflections. This principle postulates that quantum mechanics escapes from the correct measurements. For example, to calculate the position and the velocity of an electron inside the atom, we must illuminate it. But, in this way, the electron is struck by the photon and, because the Compton's effect, it changes position and velocity. If we decide to decrease the intensity of light and, as consequence, the emissions of the photons, then the velocity of electrons will be less disturbed, also if it is more difficult to identify their positions. The position and velocity of electrons cannot be simultaneously measured. A good knowledge of one of these two values presupposes the impossibility of knowing the second value. The observational act affects the observed reality, in producing inevitable interferences.[62] We don't observe the pure nature of the object, but this nature conditioned by the observational methods. The objectivity of observation is a scientific utopia.[63] At this point, each theory is lawfully adaptable to the limits imposed by objectivity. In this way, we can avoid the inconvenient questions, because if there are differences between a theory and the real world,[64] then only the theory is influenced by the limits of knowledge.[65]

Regarding the observational instruments, we can go further back in time and remember the intention of the astronomer and mathematician Friedrich Wilhelm Bessel[66] to focalize the attention on the systematic errors made by researchers during the measuring of stars position. Bessel notes that is important to consider the difference between the apparitions of the astronomical phenomena, their visual perception and the subsequent measurement of reaction done by researchers. The latency may explain the final measurement errors. During this period, several individual variations intervene as reflection of many psycho-physiological conditions (fatigue, tiredness and attention decline). Bessel

[61] Werner K. Heisenberg, "Ueber die Grundprincipien der Quantenmechanik," *Forschungen und Fortschritte* III (1927): 83.

[62] Werner K. Heisenberg, *Physics and Philosophy* (New York: Harper, 1958).

[63] Heisenberg, *Physics and Philosophy*.

[64] James B. Hartle, "Scientific Knowledge from the Perspective of Quantum Cosmology," in *Boundaries and Barriers: On the Limits to Scientific Knowledge*, eds. John L. Casti and Anders Karlqvist (Reading: Addison Wesley, 1996), 117-47.

[65] R. Rosen, "On the Limitations of Scientific Knowledge," in *Boundaries and Barriers: On the Limits to Scientific Knowledge*, 199-214.

[66] Simon Schaffer, "Astronomers Mark Time: Discipline and the Personal Equation," *Science in Context* 2, 1 (1988): 101-131.

introduces the concept of *Personal Equation of the Observer* to measure the interferences of those variables.

Exactness is another central concept of inclusive perspective. It is a quality of perfection that should crown any inductive effort of the reason. But, there are several types of induction in logic and each of them has its limits.

Associative induction, for example, is incomplete. In fact, it produces conclusions based on partial similarities between observed data. The intellect finds some common traits between two entities and so it infers the presence of another trait of similarity: if *A* and *B* are similar in an aspect (*m*), then a similarity in another aspect (*n*) is inducible. This is a shared association of elements that becomes a minimal and inconclusive generalization. This induction assumes its maximum validity only when the knowledge of data is extended, the similarities are considerable and the differences are very few.

Enumerative induction, instead, is a generalization that moves from some facts to extended conclusions. This is a pure numerical procedure that formulates certain conclusions on the basis of data that confirm them. Evidently, a certain empirical induction should be confirmed by a complete enumeration of all observable data. This condition, however, is valid only in cases of reduced sets. In other cases, instead, enumerations are incomplete and subsequent conclusions are partial and always refutable when a new fact come to contrast the facts previously observed. In these cases, enumerative induction is not certain but it is probable as well as associative induction. It is, in fact, interpretable in a statistical meaning. So, each empirical proposition assumes a hypothetical value.

Finally, causal induction goes back from known effects to their unknown causes. This procedure cannot be completely delineated. The way to know the causes can start from the observation of the constancy of some effects or from the observation of an exception in the regularity of the same effects. Also in this case, the research of causes requires a hypothetical attitude that links observations, experiments, possibility and validity. In any case, the causes may remain obscures, and so only the possible causes must be preserved.

Induction is always hypothetic both when is associative and empirical, thus probable and quantitative, and when is causal, thus possible and qualitative.[67] Knowledge shows clear empirical limits that prevent the confirming hypothesis

[67] While the analogical and enumerative inductions are based on probable hypotheses, the causal ones are based on possible hypotheses. The possibility refers to what could happen to a purely qualitative level. The probability refers, instead, to what could happen to a quantitative and statistical level.

correction. Above all, the improbable hypothesis that everything is knowable is simply unacceptable.[68]

4.2. The Foundations of Hypothetical Logic

The empirical investigation of reality falls into some paradoxes, when it is founded on objectivity and on certainty presuppositions. Detached scientific experience of the world, which transforms scientist in a cold machine, doesn't permit the perfect knowledge, but only an illusion of knowability. Observation will be always determined by what it comes from the same scientist. Observation of the world things means, primarily, bringing them into being. Even here, the boundary lines are dissipated in open horizons. The scientist, in his observational act, firstly brings and takes the same observation merged with the object of his interest. The observed reality is an interaction dimension in which we can do conjectures, speculations or real conclusions. But the risk is hidden inside this interaction. But without considering the contributions of Gestalt School regarding the illusions of perception, we find several paradoxes that arise from observation. For example, in the paradox of the solar eclipse,[69] the observer believes that is possible to see the dark side of the Moon, but what it is without light cannot be seen. Roy Sorensen[70] modifies this paradox. He invites us to imagine two different moons that make the paradox even more complex during a solar eclipse.

All knowledge is the summary of an essential coexistence often adorned with illusions and antinomies. The rigid and neutral observation cannot increase knowability and drags it to a utopian level of perfection in simply denying the existence of interactivity between object and subject. The negation of a nature and of its paradoxes causes the onset of other strong paradoxes. Frederic Fitch[71] proposes an interesting paradox of knowability. The basic idea concerns the existence of truths that are never completely knowable, although their possible conceivability. So, we can consider p as a proposition that expresses a not knowable truth and we cannot know its truth. On the contrary, p would become both a knowable and not knowable truth. Human knowledge is characterized by a certain ignorance that constitutes its premise.[72]

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[68] Cherniak, "Limits for Knowledge."

[69] Michael Clark, *Paradoxes from A to Z* (New York: Routledge, 2002).

[70] Roy A. Sorensen, "Seeing Intersecting Eclipses," *Journal of Philosophy* 96, 1 (1999): 25-49.

[71] Frederic B. Fitch, "A Logical Analysis of Some Value Concepts," *The Journal of Symbolic Logic* 28, 2 (1963): 135-42.

[72] Richard Routley, "Necessary Limits to Knowledge: Unknowable Truths." *Synthese* 173, 1 (2010): 107-22.

The weakness of inductive reasoning follows the illusion of objective observation and its subsequent knowability. Carl Hempel[73] is the creator of the famous paradox of the crows, a good example of the limitation of enumerative induction. In fact, a common generalization says that all the crows are black. However, this statement is not demonstrable in reality, because the set of crows is very dynamic and open, with a not delineable spatial and temporal extension. A real certainty is absent. Even a crow may be of a different colour, for example white, in a different historical moment or place. This is enough to confute any generalization regarding the colour of the crows. Most of the time, complete enumeration results to be impossible and generalization becomes a pure illusion. But most of the time doesn't mean always. We can, for example, admit that all books of logic, published during the 20th century, contain at least one formula, and we can evaluate the truth of this generalization. The set of logic books, unlike the one of crows, is a closed system, composed by members that are enumerable with certainty. Another paradox, created by Nelson Goodman,[74] is the 'grue and bleen' emeralds one. The uncertainty of all possible emeralds induces to adopt hybrid labels, because conceptual categories are always partial. Waiting a green emerald, because experience shows that emeralds are green, does not mean removing the possibility of the existence of a blue emerald. 'Grue' and 'bleen' concepts reveal generalizations that are valid, like any other, to describe the things of the world, but they are paradoxical because disrupt the same generalization.

Several antinomies exist also in causal induction. The paradox of donkey, created by Buridan,[75] for example, is very well known. The donkey is allowed to die when it is paralyzed by the doubt to choose between identical foods placed to its left and to its right. According to a deterministic perspective, the donkey's actions are caused by a binding condition. But, in a conflict situation, a well-defined cause doesn't exist. Identical causes cancel each other in not activating the animal's behaviour. Is the same situation valid also for humans? If we place a man between two perfectly identical tables filled with identical food, what would be his behaviour? Would he die for its uncertainty? This is, however, a remote possibility. Man seems to be free; he seems not to be bound to particular causal determinisms. However the explanation is not so simple. In fact, in a similar situation, we can consider our personal preference of the environmental side or the effect produced by a particular brightness, and so on. These are possible

[73] Hempel, "Studies in the Logic of Confirmation I," "Studies in the Logic of Confirmation II," *Aspects of Scientific Explanation.*

[74] Nelson Goodman, *Fact, Fiction, and Forecast* (Cambridge: Harvard University Press, 1955).

[75] Clark, *Paradoxes from A to Z.*

subjective causes, but still deterministic. So, determinism, as an explication of causality, is not decidable.

The predictability of future events is another idea of causal induction. If the relation between cause and effect is known, then the prevision of future would be a certainty. Also this induction falls into a paradox, the prediction paradox.[76] In fact, if we can predict what will happen in future, starting from well-known causes, then we can also be able to change the situation. But, in this way the same starting predictions would be falsified, because the future is changed.

A reality that flees knowledge is not objectively and certainly knowable. Each human experience is an interactive dimension; it is a cumulative interference of reciprocity. Reality cannot be studied with detachment. Instead, each human experience is doubtful and unpredictable. Thus, reality cannot be completely generalized. It doesn't take clear forms; it is always potential; it is full of extraordinary possibilities that leave the mind in uncertainty and openness. Each observation and each induction are only open and hypothetic.

The open hypotheses facilitate logical investigation more than others. The hypothesis is a supposition based on the available data and proposed by the researcher to be verified or falsified. The hypothesis becomes an essential part of the inductive procedure, because it shows its possible conclusions in assuming general formulations that we must prove. When the hypothesis is verified, it becomes a momentary certainty. Thus, the infinity of inductive knowledge is confined to shared and validated principles that are never certain, but always falsifiable when the new data weaken their logical strength.

5. Pure Inductivism

Some scholars[77] rediscovered free inductive logic. It is not a slave of deduction. Instead, it is based both on unlimited investigation and on intuition. The pure inductive method is focalized on the continuous research of new problems, that is a boundless and never conclusive logical system. The revaluation of investigative thought, opposed to the static truth, highlights the inexhaustible creation of problems.

Beside the mentioned inductive modalities, this logic includes abstraction and abduction procedures.[78] Even abstraction is an induction, an essential induction. This is more a philosophical than a logical modality. It is centralized on the

[76] Clark, *Paradoxes from A to Z*.

[77] Nelson Goodman, *The Structure of Appearance* (Cambridge: Harvard University Press, 1951), *Fact, Fiction, and Forecast*, *Ways of Worldmaking* (Indianapolis: Hackett, 1978), Paul K. Feyerabend, *Against Method. Outline of an Anarchistic Theory of Knowledge* (London: NLB, 1975).

[78] Carlo Cellucci, *Le ragioni della logica* (Rome-Bari: Laterza, 1998).

analysis of the basic aspects of a particular nature.[79] Abstractive induction is the capacity of reason to exclude the superficial aspects of a problem and to focalize itself on the prior resolution aspects. This is a discrimination ability, which assists any logical course with sagacity and thoroughness. Instead, abduction regards the hypothetical inferences. The hypotheses formulation explains logical problems and it characterizes rational mind that are turned to discovery and investigation.

So structured, pure inductive logic follows an uninterrupted knowledge expansion. In fact, Carlo Cellucci[80] says, pure inductive logic is:

1) Augmentative. It evolves without stopping in introducing new and progressive adjustments. It moves from the problem to not decisive hypotheses.

2) Elastic in the formulation of hypotheses. This plasticity shows the absence of the rigidity of reason.

3) Modular[81] in the contextual application of the logical domains that are able to resolve different problems. The hypothesis is part of a continuous flux of changes.

During modern times, Francis Bacon[82] represents one of the most important exponents of inductive logic. He exalts induction, because he states that only it creates solid knowledge bases, which are empirically and scientifically founded. Any scientific idea is obtained through experience. The inductive reason processes data with order, without the arbitrary abstractions of the weak conclusions; the problems are resolved and compared to their real implications. The mind that starts from empirical data is free of beliefs and of preconceptions and so it can arrive at amazing discoveries. Thanks to the use of some exclusion tables and subsidiary instances that aim to exalt the phenomenal extremisms and excesses, the mind investigates reality and it selects from the available data the most significant ones. Bacon's induction proceeds gradually from data to hypotheses, to resolutions and to knowledge growth. This course makes a non-enumerative meaning but a selective one. Induction becomes purely definitional. Bacon's thesis introduces some reflections about the limits of pure inductive logic. The research falls into purely qualitative evaluations. This defect causes the overcoming of

[79] Jacques Maritain, *Logica minore. Elementi di filosofia (II)* (Milan: Massimo, 1990). Juan J. Sanguineti, *Logica Gnoseologia* (Rome: Urbaniana University Press, 1983), *Logica filosofica* (Florence: Le Monnier, 1987).

[80] Cellucci, *Le ragioni della logica*.

[81] Types of modularity: 1) cooperation (compatibility between modules); 2) pre-emption (introduction of innovative modules); 3) negotiation (compromises between modules) (Cellucci, *Le ragioni della logica*).

[82] Francis Bacon, *Nuovo Organo* (Milan: Rusconi, 1998).

Bacon's logic in a historical period, between the 16th and the 17th centuries, signed by the birth of Galilean deductive method.[83] The need to give substance to knowledge deprives Bacon's method of real quantitative implications, so it becomes weak and uncertain in its conclusions. Pure inductive method results are poor and doubtful.

Instead, James Stuart Mill[84] pays attention to conceptual elaboration of some causal inductive methods and he prefers to study the differences, concordances and connections that occur between causes and effects. Thanks to Mill, pure inductive logic further shows its limits of formalization, because it becomes purely possible.[85] But, if this feature may not be a real constraint, then Mill's methods are surely weaker when they are applied to complex reality as, for example, social or psychical ones. Above all, this inductive procedure shows several difficulties in understanding the relevant causes[86] and it reproduces a sort of logic linearity that moves from problem to possible resolutions without deviations. This is an ingenuous logic because it doesn't represent the multi-linearity and the circularity of causal connections.

When inductive research becomes boundless, logical inexhaustibility doesn't have impositions and it dissipates knowledge in a confusing vanishing. Human knowledge must not encounter arrests and condensations; it must not languish in rigid stages, but it must lose itself in an uncurbed, relativistic and steamy growth. Any acquired knowledge becomes partial and so it assumes the characteristics of a despotic illusion, when it is considered very reliable. The truth is always adaptable to contingences and it is never absolute. The research shows the meaning of an investigative and indubitable instability. The infinity in progress dominates the logical outlook. Knowledge flows toward inexhaustibility and hypothetical method plays a central role on it. An unstoppable investigation process takes form. Each hypothesis becomes part of an unlimited hierarchy and of a continuity that, idealistically, starts from a problem and covers all human history.[87] From a relativistic point of view, knowledge becomes a constructive force that forms, through induction, some reality maps and this facilitates the creation of a purely subjective world.[88]

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[83] Alfred N. Whitehead, *Science and the Modern World* (New York: Free Press, 1997).

[84] James Stuart Mill, *A System of Logic: Ratiocinative and Inductive* (Ithaca: Cornell University Library, 2002).

[85] Irving M. Copi, Carl Cohen, *Introduction to Logic* (Upper Saddle River: Prentice Hall, 2008).

[86] Copi, Cohen, *Introduction to Logic*.

[87] $Ip_1 \rightarrow Ip_2 \rightarrow Ip_3 \rightarrow Ip_4 \rightarrow \ldots \rightarrow Ip_n$. With $n=\infty$.

[88] Goodman, *The Structure of Appearance*, *Fact, Fiction, and Forecast*, *Ways of Worldmaking*.

In *Pure Inductivism*, the hypothetical concatenation is infinite. The first hypothesis (*Ip1*) is not true if the second (*Ip2*) is false, and this hypothesis is not true if the third (*Ip3*) is false, and so on in an infinite regression. Hypothetical concatenation has no explicative value because everything flows in a linearity of conditionals. In this wild hypothetical method, no hypothesis is true if the conclusive hypothesis of the chain is false. However, this final hypothesis coincides with the infinity, which represents the necessary but not sufficient condition of all hypothetical inductive concatenation.[89] However, if we pose the true limit of the chain in a hypothetical infinite condition, then we establish a sort of paradoxical demonstrative self-reference that is clearly opposed to the open idea, typical of this logical model. If the infinity confirms the infinite hypotheses, then the circularity is evident and the relativism of knowledge becomes dominant.

Paul Feyerabend[90] expresses the peak of this relativism. In his ruthless attack against philosophy of science, considered useless or even parasitic, Feyerabend believes in a science that must enjoy of an absolute and anarchist freedom. Science should not have constraints, not even an illusory authority like reason. Feyerabend underlines that, at times of maximum scientific growth, figures such as Galileo came to their discoveries taking advantage on non-scientific capacities such as fantasy, cunning, rhetoric and propaganda. Without the silence of reason in the most important circumstances, science would grow less. Feyerabend concludes that inventiveness and creativity should not be inhibited.

5.1. The Realistic limits of Pure Inductivism and the Contained Logic

Pure inductive logic exalts the infinity of research and of human knowledge. This idea seems persuasive: it causes openness and freedom. Above all, it represents an important reaction to closures and contradictions of deductive logic. But it presents several limits.

The infinity misses the cumulative aspect of human knowledge. A not conclusive infinity does not give real knowledge. It becomes a confusing and vain research. Also the several promoters of pure inductive logic often remark the importance of the acquired knowledge and they underline that rational investigation must always adapt its courses to it.[91] Any acquired knowledge, however, presupposes a basilar and augmentative quiescence. The infinity is, thus,

[89] Peter D. Klein, "Human Knowledge and the Infinite Regress of Reasons." *Philosophical Studies* 134, 1 (2007): 1-17.

[90] Feyerabend, *Against Method*.

[91] Cellucci, *Le ragioni della logica*.

in the evolution of knowledge, in the conservation of some traits, in the change of other traits and in the general mutability. Each scientific progress is a profitable union of advances, arrests, involutions and unstoppable cognitive movements. Watching the history of human thought, we can discover that several theories persist for centuries, although changed or reduced. At the same time, we can discover that the other theories were buried by progress. Nevertheless, when there was a new discovery, a common knowledge base was present. Nothing of new arises without that something old is first widely accepted.

Some certainties seem to exist, but they are not useful to adopt an axiomatic logic. The idea of infinity must not deny the certainties, but it simply have to keep them in doubt because their future falsification is always possible.[92] However, during their existence, these certainties regulate any investigation and any formalization, thus they hold their same possible confutation. Idealistically, these certainties could always be valid because, like any other knowledge, they enjoy of infinity, which logically maintains them in doubt. Any certainty can be truthful, useful, fascinating and all comprehensive, but never immovable.

Pure inductive method considers hypotheses as methodological foundations of investigations, but it is a controversial method. Each question and each problem are reduced, in fact, to an only possible resolvable condition, so that referring to infinity presupposes a sort of intrinsic irresolution of the natural dilemmas. Instead, the hypotheses can capture important knowledge, established and accepted in their truthfulness, which persist over time in signing a continuative lull that leans towards the logical infinity, with the possibility to have an infinite lull. Resolvability becomes a journey of discovery where the formulated hypotheses can be provisory accepted or imperatively refuted or, sometimes, indefinitely preserved over the time. When obtained resolvability is high and effective, nothing is against the possibility to perpetuate the derived knowledge. But the primary hypothetical nature remains unchanged. Everything can be reviewed and refuted, but everything may remain unaltered. The hypotheses are the results of an active research. They are creative products that increase the growth of knowledge, maintain a link with the previous learning and impose themselves as evolutionary and non-invertible steps. So, we cannot say that any hypothesis resolves only specific problems, so as the exponents pure induction sustain.[93] The development of knowledge predisposes any discovery to more or less stable generalizations. The resolvability of a problem requires other similar problems. So, if knowledge is partial and incomplete in any specific moment, then these

[92] Karl R. Popper, *The Logic of Scientific Discovery* (London: Routledge, 2002).

[93] Cellucci, *Le ragioni della logica*.

partiality and incompleteness decrease during the growth of knowledge. In an indefinite time, knowledge will become complete.

Thus, inductive logic is primarily an open mental attitude with a particular preference for what is inexhaustible, but it is also a conviction that nothing limits knowledge, not even the same infinity that takes on all axiomatization traits, when it is intended as a necessity. Therefore, as well as *Strong* and *Weak Deductivism*, *Pure Inductivism* is a closed logic.

5.2. The Foundations of Contained Logic

The idea of infinity does not escape, as the other logical ideas, from the power of paradox. If, for example, we compare the unlimited sequence of all natural numbers (*1, 2, 3, 4, 5,…, n*) with the sequence of the even numbers (*2, 4, 6, 8, 10,…, n*), then we can think that an infinite set corresponds to another infinite set that seems to be exactly its half. We can think that the infinite set of all natural numbers is more infinite than the infinite set of all even numbers. We can introduce a further idea of differentiation between infinity and numerousness of a sequence: while several numerical sequences can be equally infinite, some of these may be more numerous. The idea of infinity is extended by another specific term. Galileo solves this paradox.[94] In fact, he couples the sequence of double numbers (*2, 4, 6, 8, 10,…, 2n*) with the infinite sequence of natural numbers (*1, 2, 3, 4, 5,…, n*), in knowing that a double number is always an even number. Although we can consider the first sequence more numerous than the second sequence, every natural number can be paired one-to-one with a specific even number. Georg Cantor[95] creates a contemporary version of Galileo's paradox. Cantor's antinomy refers to the idea that each set is always strictly smaller than its power set,[96] which represents the class of all its sub-sets. Also the infinite set is small if it is compared with its power set.

The antinomies of nfinity date back to oldest times, when their narrative aspect was predominant. One of the most famous of them is the paradox of Achilles and tortoise described by Zeno of Elea.[97] Achilles decides to challenge a tortoise to a race but, being much faster, gives it an advantage. While he starts from the point p_1, the tortoise starts from the point p_2. But when Achilles reaches the point p_2, the tortoise is already at the point p_3, and when Achilles reaches the point p_3, the

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[94] Clark, *Paradoxes from A to Z*, Luminet, Lachiéze-Rey, *De l'infini…*

[95] Georg Cantor, *La formazione della Teoria degli Insiemi* (Florence: Sansoni, 2002), *Contributions to the Founding of the Theory of Transfinite Numbers* (New York: Dover, 2010).

[96] *X* is a set *{1,2,3}*. *P(x)* is the power-set of *X*. *P(x)* contains the empty set *{∅}* and the sets *{1}*, *{2}*, *{3}*, *{1,2}*, *{2,3}*, *{1,3}*, *{1,2,3}*.

[97] Clark, *Paradoxes from A to Z*.

tortoise is already at the point p_4, and so on ad infinitum. Whenever Achilles covers the new distance, the tortoise has already moved outdistancing him of a smaller and smaller stretch of road. The intervals that Achilles must cover become infinite. The stadium paradox of Zeno and the paradox of gods are very similar to that of Achilles and tortoise. In the first of these,[98] Achilles must run on the stadium track. But he cannot cross the finish line because he should cover a half of the remaining distance every time, but each section of the trail has its own half. Instead, in the paradox of gods described by Josè Benardete[99] a man wants to walk a mile. However, an infinite number of gods interferes with his path. When the man will reach a half of mile, one of these gods will intervene to put an obstacle. Another god will put, instead, a similar obstacle when the man will reach a quarter of mile, and another god will put it at an eight of mile, and so on ad infinitum. The man will be overwhelmed by the immensity of the obstacles, and so he cannot move from its starting point. But, finally, there will be no obstacles because the man does not move. He remains motionless simply because the gods' intentions will not be realized.

In conclusion, infinity is a persuasive idea and its direct experience is impossible. A limited condition, which can be existential, psychological or cognitive, makes man restless. A sense of unease reveals the utopias of perfection and certainty. An intuition vividly grows: something always escapes from these fatuous and illusory boundaries. Just intuition is, in fact, a basis of the idea of infinity. Finiteness is proper to human condition, but cosmos is not finite; it always extends its perimeters. But, when an idea derives from its original sources, it already forces and defines, in closing cosmos in boundaries. The idea of infinity, just like any other idea, occludes its own real content. In this way, the same idea distorts and betrays the indefinability and boundlessness of the original meaning. Infinity refers to what is always beyond human comprehension, so any attempt to think about it and to express it is a misrepresentation. This is the first fundamental contradiction of infinity. The idea of infinity forces to contain what is uncontainable second nature. It is not evidence and so it is inexpressible and it is indicative of an elusive quality. The second fundamental contradiction of infinity is its implicit lack of conformity with concrete reality. Experience is limited and limiting, although never conclusive.

[98] Clark, *Paradoxes from A to Z*.

[99] Josè A. Benardete, *Infinity: An Essay in Metaphysics* (Oxford: Clarendon, 1964).

6. The Logical Limits of Scientific Knowledge

At this point, we can propose two questions: 1) How many are the logical limits of knowledge? 2) What are these limits? We can answer simply retracing our present work, but we must premise that a list of limits could never be complete. Furthermore, the Princeton Group (Piet Hut, David P. Ruelle, and Joseph F. Traub)[100] proposed an interesting list of the limits of knowledge: 1) curse of the exponential (some problems, such as chaos, can be solved in principle, but they are intolerably hard); 2) asking the wrong questions (there are structural limitations to the questions one may ask because some of these have no answers, they should not be asked, such as asking to specify the position and velocity of a quantum particle in a specific moment); 3) questions of questionable status (it is always unclear if a natural question that appears hard to answer corresponds to a fundamental limitation or to a bad problem); 4) emergent properties (the study and understanding of the higher levels of reality are very difficult); 5) limited access to data (in some areas of science, as the historical ones, the absence of sufficient data leads to severe limitations); 6) sample size of one (in sciences that deal with a single dimension, as cosmology, that studies the origins and structure of the universe, it is difficult to compare theories with observations); 7) technological limitations (some important limits affect the scientific practice, but the limits that can be overcome by new ideas are no real limits).

The logical form is common to all sciences, although it is different in its subjective manifestations, and so we can argue that each limit of scientific knowledge can be read in terms of logical limits, and vice versa. In fact, they are different terms that denote the same thing. According to our point of view, there are some extensive categories of limits that include the limits of Princeton's list:

1) Excess of logical rigor and categorical closures. The attempt to create complete and coherent knowledge systems collides with limitlessness of knowledge, so revealing the limits of demonstrative logic. In this category, we include 'asking the wrong questions,' because the fact that a question must not escape from demonstration criteria reveals its strong limitation, and 'questions of questionable status,' because several theories are not really reliable and complete.
2) Complementarity between objective and subjective worlds. The illusion of understanding reality, without considering human presence, collides with the complexity of cosmos, in revealing the limits of observational logic. In this

[100] Piet Hut, David P. Ruelle, and Joseph F. Traub, "Varieties of Limits to Scientific Knowledge," *Complexity* 5 (1998): 33-8.

category, we include 'limited access to data,' because all infinite data are not observable and collectible, 'technological limitations,' because the instrument needs continuous adjustments that follow the progress of knowledge, and 'sample size of one,' because in some sciences the comparison between theories and observations is very difficult.

3) Claims of hypothetical investigation. The intention to obtain certainty collides with the never decisive nature of the investigation, so revealing the limits of investigative logic. In this category, we include 'curse of the exponential' and 'emergent properties,' because some problems remain unsolvable at the current state of knowledge.
4) Dazzling inexhaustibility of hypothetical investigation. Infinity collides with its idea, so revealing the limits of the fatuous and ephemeral of scientific research. This category is not considered in Princeton's list and so it enhances it with a new element.

7. Some Considerations about the Changes and the Revolutions of Scientific Knowledge

In the course of our present work, we considered the main historical logical forms of scientific knowledge. Each of these forms, however, has shown some specific limits that have triggered revolutionary changes, when they became aware in scientific community. But, in talking about science revolution, we must compare our work with the ideas of Thomas Kuhn.[101] In fact, he also speaks about scientific revolution but he introduces some different concepts. Therefore, we believe that is necessary to develop a critical and integrative work. According to Kuhn, scientific knowledge oscillates between phases of stability and phases of change and revolution, which are announced by more or less deep crisis. During the phases of stability, or phases of normal science, scientific knowledge is structured around one or few basic rigid paradigms.

In a general meaning, the term 'paradigm' refers to the concept of "disciplinary matrix" that is the basic set of beliefs shared by a group of scientists.[102] According to the gestaltist perspective of Kuhn, the paradigm influences the perception of reality, and its subsequent interpretation. Moreover, because of different cultural and psychological conditions, in certain periods, these

[101] Thomas S. Kuhn, *The Structure of Scientific Revolutions (1962)* (Chicago: University of Chicago Press, 1970).

[102] Kuhn lists some beliefs that constitute the disciplinary matrix: 1) the symbolic generalizations; 2) the metaphysical paradigms or the metaphysical parts of paradigms, which are philosophical metaphors; 3) the scientific values; 4) the solutions of the typical problems.

paradigms highlight anomalies that, until then, had remained implicit or were considered not significant. When some authors explain these anomalies, they can trigger changes and revolutions. So, the paradigm changes drastically or it is replaced by another innovative paradigm. During the transition from the normal science to the revolutionary science, which will become normal in time, the paradigm is not rigid but changeable: this is a sort of pre-paradigmatic phase.

The common interest in scientific change allows a first and interesting correlation between the concept of 'paradigm,' which is fundamental in the study of Kuhn, and the concept of 'logical form,' which is instead basic in our present study. The two concepts are different by definition. While, in fact, the logical form is the rational style of scientific knowledge, the paradigm is instead its point of view on the world. Therefore, scientific knowledge constitutes their common nature. These two concepts, although different by definition, have also significant aspects of similarity in their common tendency to change, because all scientific knowledge is mutable. Indeed, we can argue that: 1) 'logical form' and 'paradigm' are both limited and ready to future processes of change; 2) these limits remain implicit until the scientific community take aware of them; 3) while the logical limits emerge as contradictions and inconsistencies, paradigmatic limits emerge as anomalies; 4) the change may eventually lead to a real scientific revolution that will be logical or paradigmatic. The paradigm and logical form are strongly linked, because their nature is common and their evolutions are similar. In fact, we think that scientific knowledge is logically structured around its paradigms; it processes the theoretical and methodological implications of its beliefs through a specific rational style. The same Kuhn, who relegates logic to a not significant component of paradigmatic change processing, implicitly identifies the logical form of each paradigm. In fact, in addition to the general definition of the term, he presents a more specific definition that, however, not too subtly conceals its logical sense. According to Kuhn, in starting from the usual problem, the scientist learns to see different situations as similar to each other, as subjects to the application of the same law or draft law. This learning is not verbal, but practical, because it resulted from the exercise. The scientist learns to recognize the similarity or dissimilarity between different examples and exercises. This definition, however, can be easily presented in logical terms. In fact, similarity relations are at the base of both the applications of deductive theories to multiply concrete examples and inductive generalizations of the analogy type: similar examples for the same theory in the first case; similar examples for the same generalization in the second case. We can therefore argue that each paradigm binds to a specific logical form that is deductive or inductive.

Given the link between the logical form and paradigm, we can also better explain the relations between the logical and the paradigmatic limits. In fact, no change is possible without a limit to exceed. The logical limits manifest themselves by paradoxes and structural antinomies, the paradigmatic limits by functional faults. In this regard, however, the historical study of science underlines an interesting and explaining characteristic: the emerging of the logical and paradigmatic limits, as well as their subsequent processes of change, may not always coincide and happen separately in different periods. So there are three possible conditions of change: 1) Only the logical limit emerges that starts a changing process or a revolution of the corresponding logical form; 2) Only the paradigmatic limit emerges that starts a changing process or a revolution of the corresponding paradigm; 3) Both of the limits emerge, at the same time or in different phases, which start a global change of the logical forms and their correspondents paradigms.

We can mention, as historical example of the first condition of change, the development of the third phase of Cognitivism, the Emergentism phase,[103] that took place between the eighties and the nineties of the last century. During these years, the American cognitivistic psychologists take awareness of the logical limits of their discipline, at that time divided into two approaches: 1) Computationalism, born in the first phase of Cognitivism, intends the mind as an highly symbolic cognitive dimension; 2) Connectionism, born in the second phase of the Cognitivism, on the other hand, intends the mind as a sub-symbolic neuronal dimension. According to Emergentism, Computationalism is too abstract while Connectionism is empty of any abstraction. The exponents of the rising Emergentism[104] want to integrate the two models into a single causal model that is, at the same time, top-down (cognition, as macroscopic dimension of the mind, causes neurological effects) and bottom-up (neurology, as microscopic dimension of the mind, causes cognitive effects). Mutual causing constraints are impressed between these two levels.[105] So the neuronal level is meant as a basic level from which it differs a higher level, with its own distinctive characteristics, which is

[103] The notion of emergence implies the notion of novelty.

[104] The rise of this new phase is due to skills theorists. They study a type of learning not new but very complex, which is difficult to explain by the traditional models. The motor learning implies, in fact, very articulate behaviours that are hierarchical, non-linear and non-sequential (Karl S. Lashley, "The problem of serial order in behaviour," in *Cerebral Mechanisms in Behavior*, ed. Lloyd A. Jeffress (New York: Wiley, 1951), 112-146. There are in fact many variables that cannot be controlled (Nikolai A. Bernstein, *The Coordination and Regulation of Movements* (Oxford, New York: Pergamon Press, 1967)).

[105] Bechtel, *Philosophy of Science*.

the symbolic level of pure cognition. It places a bridge between biology and the psyche. The hierarchy represents an emersion, from the basic levels, of other levels more and more complex. The new Emergentism is a revolution in the logic of Cognitive Psychology, but not in its paradigms. The point of view on the mind is not changed: the mind maintains its psychic centrality, and Emergentism is not totally a new theory, but it is a summary of the integration between Computationalism and Connectionism. However, the basilar logical form is changed significantly. While Computationalism and Connectionism highlight a logical form of Weak Deductivism (empiricism serving theory), Emergentism highlights a logical form of Pure Inductivism (empiricism as an axiom of inexhaustibility). In the Emergentist phase, in fact, takes place the prospective of a definition of hierarchical horizons anchored to empirical and organic data that could be infinitary from an intuitive point of view.

As historical example of the second condition of change, we can mention the transition from Euclidean Geometry to non-Euclidean Geometries. Carl Friedrich Gauss's school is at the origin of this paradigmatic revolution.[106] He recognizes the impossibility to prove Euclid's fifth postulate[107] and he convinces himself about the legitimacy of to build up a coherent geometric system based on its own negation. Gauss's new Geometry reflects the proprieties of the space that are contradictory only in appearance. Janos Bolyai[108] and Nikolai Lobachevski[109] formalize the first real model of non-Euclidean Geometry. They proposed, similarly, a hyperbolic geometry. This geometry is based on the hypothesis of the acute angle of Giovanni Girolamo Saccheri[110] or Johann Heinrich Lambert, and establishes that for a point outside a straight line, it is possible to conduct only two parallel lines. In the second half of the nineteenth century, Bernhard Riemann postulated a second form of elliptic geometry.[111] This geometry, based on Saccheri's hypothesis of the acute angle, establishes the non-existence of the

[106] Harold S.M. Coxeter, *Non-Euclidean Geometry* (Whashington, D.C.: Mathematical Association of America, 1998).

[107] The Fifth Postulate, which is unintuitive, establishes that, given a straight line and a point external to it, one and only one other straight line, that is parallel to it, passes through that point.

[108] Janos Bolyai, *The Science Absolute of Space; Independent of the Truth or Falsity of Euclid's Axiom XI (which can never be decided a priori)* (University of Michigan: University Library, 2005).

[109] Nikolai I. Lobachevski, *Geometrical Research on the Theory of Parallels*, trans. George B. Halsted (New York: Cosimo, Inc. 2007).

[110] Giovanni Girolamo Saccheri, "Euclid Freed From Every Flaw (excerpts)," in *A Source Book in Mathematics*, ed. David E. Smith (New York: Dover Publications, 1959), 351-359.

[111] Coxeter, *Non-Euclidean Geometry.*

same parallel lines. Compared to Euclidean Geometry, non-Euclidean Geometries represent a change of paradigm, because the axioms change deeply, but the logical form of Strong Deductivism does not change.

As example of the third scientific change, we can also mention the rise of Darwinian Evolutionism that represents, in the nineteenth century, both a logical and paradigmatic revolution of science. Evolutionism defines the world as a set of constantly augmentative phenomena.[112] In fact, the concept of continuity represents the base of the new paradigm. Animal species constantly evolve to biological and mental forms always more functional and adaptive. The implicit inexhaustibility of the evolutionistic paradigm hides his logical inductive form, but science till the nineteenth century was not at all inductive. The revolution triggered by Evolutionism is therefore both logical and paradigmatic. This revolution proposes a sense of inexhaustibility that sensitizes the mind to philosophical Relativism. Also the rise of the Quantum Physics represents an example of this third condition of scientific revolution. The new paradigm introduced transforms, as a matter of fact, the idea of the sub-atomic world: Quantum Physics removes the previous distinction between particles and waves. A quantum system has the typical characteristics of the waves, but when it is measured, or even observed, takes on the characteristics of a set of particles (quanta). The new logical form abandons the old deductivisms, still dominant in Einstein's physics, to embrace a probabilistic perspective that is, instead, very indefinite and inductivist.

We present a final important difference between logical and paradigmatic revolutions of science. The paradigm, in fact, is a set of beliefs that is conditioned by historical, cultural and psychological contingencies. The logical form, instead, is a pure rational structure, less influenced by such contingencies. The history of science confirms this idea: while the paradigmatic changes and revolutions are the effect of some alternative tensions and tendencies that are increasingly evident in some historical period, the logical changes are instead the effect of the intuitions of some brilliant scholars. In fact, thinking back to the logical revolution of Gödel, we can see that it causes the collapse of Formalism in a time when it was strong and vigorous. Moreover, Gödel had no intention of demolishing the finitist illusion of Formalism. Historically, Formalism was not ready to die.

The work of Thomas Kuhn is a very important work, because of its insistence on the role of paradigms, but it is also objectionable, because it

[112] Pavel A. Florenskij, "Su un presupposto della concezione del mondo," in *Pavel A. Florenskij. Il simbolo e la forma*, eds. Natalino Valentini and Alexandre Gorelov (Turin: Bollati Borighieri, 2007), 13-24.

underestimates the role of reason in the development and the progress of science. Similarly to us, Imre Lakatos[113] recognizes the centrality of reason in each scientific revolution. According to him, if what he calls "research programme" of science is progressive, then reason supports knowledge and it solves and integrates the paradigmatic anomalies. If, instead, this programme is degenerative, then reason facilitates the paradigmatic change or the revolution.

We finally mention another important feature of the logical revolution: the immediate consequence of every great discovery. In moments of great discoveries, a sense of openness and innovation pervades science. But soon this 'spirit' becomes less intense, when a new and dominant logical form becomes more stable. When a logical form is stable it is also closed, but when the revolution is coming it can become open. A typical example is the birth of paraconsistent logics that we will analyze in the paragraph below, also for other reasons.

8. The Limits of Thought in the Innovative Tendencies of Logic

We can now consider some contemporary logical tendencies that constitute good examples of scientific changes and revolutions. We can take the example of the new paraconsistent logics. These tendencies also study the limits of thought, but with the intention to confirm their theoretical and methodological positions, in overshadowing the specific importance of the theme. Paraconsistent logics consider the contradictions as opportunities to extend the paradigm of logics. Some scholars[114] believe that several contradictions of thought are true. Thought uses its limits to access to alternative but true logical worlds, where the paradigms of classical logics collapse. More specifically, this logic abolishes the law of non-contradiction, so that everything can be true and false at the same time. In this way, paraconsistent logics aim to Logical Relativism, where it is possible to say everything and the opposite of everything, i.e. nothing. Paraconsistent logics are specific scientific knowledge, and therefore they take a specific logical form that is of Pure Inductivism.

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[113] Imre Lakatos and Alan Musgrave, eds. *Criticism and the Growth of Knowledge* (Cambridge: Cambridge University Press, 1970).

[114] Graham Priest, Richard Routley and Jean Norman, eds. *Paraconsistent Logic: Essays on the Inconsistent* (München: Philosophia Verlag, 1989), Graham Priest, *Beyond the Limits of Thought* (Cambridge: Cambridge University Press, 1995), "Paraconsistent Logic," in *Handbook of Philosophical Logic*, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Kluwer Academic Publishers, 2002), 287-393.

Graham Priest[115] explains the legitimacy of paraconsistent logic in considering the limits of thought. In fact, these limits are occasions of illogicality, where thought is free from its rational constraints, so being able to create new and stronger logical systems. However, Priest unwittingly confuses the logical plan with the paradigmatic one, but he implicitly remembers to us the inseparability of the two plans. In fact, he defines four specific limits that we can present in logical and paradigmatic terms: 1) Limits of expressible. The features of the world transcend the ability of language to express them. Each point of view on the world is so limited. But in saying what those features are, we are liable to say the unsayable, and this is an evident logical contradiction. 2) Limits of iterable. Some operations are applied over and over again as far as possible. A representation of the world is constantly being proposed. The paradox of the mathematical infinity is typical: though there be no greater than the infinite, but there be a greater. But the paradoxes of the infinity are clearly logic. 3) Limits of conception. There are things beyond conception. Each point of view on the world may represent only a few things. But it is difficult to do so without conceiving them in some sense. Hence the logical contradiction at the limit of conception. 4) Limits of cognition. Several relationships arise between agents and the world that they cognise, between thought and the states of the world. However, at the same time, several limits arise as anomalies between representations and the things observed.

9. Conclusions

We think that a good closure of our argumentations has to say something about the possibility of solving and exceeding the limits of scientific knowledge. We believe that these limits provide an essential part of this knowledge. Some philosophers think, as happened in the case of David Hilbert and his school, that science is based on the idea of logical closure. Other philosophers think instead that science is based on the idea of perfection of its methods (as happened in the case of the members of the Vienna Circle). Finally, in the works of other philosophers, e.g. Paul Feyerabend, science loses itself in indefinable and relativistic perspectives. But every time, some brilliant scholar reveals the illusory that is hidden in these perspectives. In this regard, we can remember the works of Kurt Gödel, alan Turing, Werner Heisenberg, Bertrand Russell, and so on. In such works, scientific knowledge shows its real face. It shows that it cannot be closed, because it is open; that it cannot be completely open, because it contains; that it cannot be certain, because it is hypothetic.

[115] Priest, *Beyond the Limits of Thought.*

At this point, we can essentially define science as a form of knowledge that is characterized by opening, possibility and containment. However, an essential definition of scientific knowledge requires an integrated definition of its characteristics. In this way, the risk of contradictions and paradoxes is avoided, as first of all the self-reference of terms: 1) opening is, at the same time, contained and possible; 2) containment is, at the same time, opened and possible; 3) possibility is, at the same time, contained and opened. Revolutions and stasis, reactions and innovations, creations and rationalizations are aspects of a wonderful human experience: science, whose weakening may cause limitations and contradictions.

THE BADNESS OF BEING CERTAIN OF A FALSEHOOD IS AT LEAST 1/(log 4 – 1) TIMES GREATER THAN THE VALUE OF BEING CERTAIN OF A TRUTH

Alexander R. PRUSS

ABSTRACT: Surprisingly precise results are provided on how much more one should disvalue being wrong than one values being right.

KEYWORDS: credence, epistemic utility, probability

1. Introduction

William James famously thought that epistemic agents differ in how much they comparatively hate error and love truth. Some people "regard the chase for truth as paramount, and the avoidance of error as secondary; or we may, on the other hand, treat the avoidance of error as more imperative, and let truth take its chance."[1] He thought that being a person of the second sort was reasonable. However, it seems that he was wrong. On reasonable assumptions, and bracketing non-epistemic utility considerations, we can show that a rational agent should 'hate' or disvalue being certain of p if p is false at least $1/(\log 4 - 1) \approx 2.588$ times as much as she 'loves' or values being certain of p if p is true. More generally, if $r \geq 1/2$, one should 'hate' having credence r in p when p is false at least $(2r-1)/(1 - 2r + \log 4 + 2 \log r)$ times as much as one 'loves' having credence r in p. For instance, you should 'hate' assigning credence 0.95 to a falsehood more than 2.345 times as much as you 'love' assigning credence 0.95 to a truth.

It is surprising that such precise results can be obtained. They will be obtained as a corollary of a necessary condition on proper concave epistemic utility functions.

Normally, epistemic utility functions measure the epistemic value of one's credences given what the truth of the matter is. In this paper, our focus will be on the epistemic utility of one's credence in a *single* proposition p, however, rather than the epistemic utility of one's epistemic state as a whole. This is all that is

[1] William James, "The Will to Believe," *The New World* 5 (1896): 327–347.

needed for the results about love of truth and hatred of error that are announced in the introduction and it simplifies the notation while focusing us on the essentials.

2. Proper epistemic utility functions

Throughout, fix a proposition p of interest. We can measure the epistemic utility of a credence r in p by a pair of functions. $U_T(r)$ is the utility of having credence r in p should p be true. $U_F(r)$ is the utility of having credence r in p should p be false. These functions measure how much one 'loves' or 'hates' being right or wrong about p. We shall allow U_T and U_F to take either finite or infinite values at extreme points. Our main interest is in the case where $r \geq 1/2$.

Normally, scoring rule analyses work in terms of measures of *inaccuracy*– the greater the number, the worse. We shall formulate the results in terms of utilities in order to fit with the value-based considerations driving the analysis, and we shall do so in such a way that a familiarity with the scoring-rule literature is not required in the reader. It is worth noting that the above setting is somewhat more general than typical scoring-rule analyses as it allows that the utility-if-true and utility-if-false functions can differ depending on the proposition p in question. Our claims are always about a single proposition p.

We will now impose some reasonable conditions on U_T and U_F. The first constraint is uncontroversial:

(a) The function U_T is monotonically increasing and the function U_F is monotonically decreasing.

Our next condition is:

(b) The functions U_T and U_F are continuous on the interval [1/2,1], differentiable on its interior (1/2,1) and finite-valued on [1/2,1).

This assumption could be weakened, but it will make the mathematics more convenient.

The following constraint is a weaker version of a fairly standard, though controversial, assumption about scoring rules:[2]

[2] James M. Joyce, "A Non-Pragmatic Vindication of Probabilism," *Philosophy of Science* 65 (1998): 575-603, James M. Joyce, "Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial belief," in *Degrees of Belief*, eds. F. Huber and C. Schmidt-Petri (Dordrecht: Springer, 2009), 263-297.

(c) The function U_F is concave on [1/2,1].

A continuous function f is concave on an interval I provided that $f((a+b)/2) \geq (f(a)+f(b))/2$ for all a and b in the domain.[3] Our concavity assumption (c) parallels a standard but controversial convexity assumption on scoring rules (utilities increase with better match between credences and truth, while scores decrease with better match), but it is weaker than that assumption by being restricted to the case where one assigns credence $r \geq 1/2$ to a falsehood.

The concavity assumption (c) is in fact quite plausible given the restriction. Intuitively, if p is false, you lose more – or at least no less – by a fixed increase of credence the closer your credence is to 1. Thus, an increase in credence from 0.50 to 0.55 is mildly unfortunate, an increase from 0.55 to 0.60 is no better and very likely worse, an increase from 0.60 to 0.65 is still no better and very likely worse, and so on, until the extremely unfortunate increase in disutility from 0.95 to 1 when you become *certain* of a falsehood. Generalizing this reasoning to all increments implies the concavity of U_F.[4]

Alternately, one might argue like this. Suppose p is false. It intuitively takes a stronger piece of evidence to return one from credence 0.95 to credence 0.90 than to return one from credence 0.90 to credence 0.85, and so on. Therefore, the loss of epistemic utility in moving from 0.90 to 0.95 is greater than in moving from 0.85 to 0.90, because it is harder to return.

Intuitions might be divided on whether U_F is concave on the full interval [0,1]. I am inclined to think it shouldn't be taken to be concave on [0,1/2]. If it were concave, then the gain in utility in a transition from, say, credence 0.25 in a falsehood p to a credence 0.15 in that falsehood would be at least as great as the gain in utility in a transition from 0.50 to 0.40. But the latter seems a more significant transition: it is a move from being on the fence to having a significant inclination to the truth.

The final constraint is the crucial one:

(d) The pair (U_T, U_F) is proper.

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[3] In general, if we have no continuity assumption, to define concavity we need to say that f(αa+(1–α)b)≥αf(a)+(1–α)f(b) for all a and b in its domain and all α€(0,1).

[4] Suppose a and b are in [1/2,1], and suppose for simplicity that $a<b$. Let $\delta=(b-a)/2$ and let $c=(a+b)/2$. Then $c \geq a$ and the generalized claim in the text implies that $-U_F(c+\delta)-(-U_F(c)) \geq -U_F(a+\delta)-(-U_F(a))$, because $-U_F(r)$ is the *disutility* of having credence r in the falsehood p. But $c+\delta=b$ and $a+\delta=c$, so $-U_F(b)+U_F(c) \geq -U_F(c)+U_F(a)$, from which it follows that $2U_F(c) \geq U_F(a)+U_F(b)$ and so $U_F((a+b)/2) \geq (U_F(a)+U_F(b))/2$, which implies concavity.

A pair of utility functions is *proper*, roughly speaking, just in case it is never decision-theoretically rational, in respect of the epistemic utility of these functions, to change one's credences without any further evidence.[5]

Before giving a formal characterization of propriety, we can give an example of an *improper* pair of utility functions, and explain why it is improper. It may seem initially very plausible to choose the linear functions $U_T(r)=r-1/2$ and $U_F(r)=1/2-r$. But this has would have untoward consequences. Suppose p is the proposition that a toss of a fair six-sided die will *not* yield 6. Obviously, my credence in p should be 5/6. But consider the expected epistemic utility of having credence 5/6 in the die toss. I have probability 5/6 of being right and 1/6 of being wrong. My expected epistemic utility, then, is $(5/6)U_T(5/6)+(1/6)U_F(5/6)=2/9$. But what if I just go out on a limb and am *certain* that the toss won't yield 6? My expected epistemic utility, then, is $(5/6)U_T(1)+(1/6)U_F(1)=1/3$. And $1/3>2/9$. More precisely, one can easily check (say, by drawing a graph) that the expected epistemic utility maximizing credence in this case is 1. But it's perverse to switch one's credence from 5/6 to 1 in this case, and any pair of utility functions that recommends it is perverse, or at least *improper*.[6]

Formally, we say that the pair is proper provided that for each r in $[0,1]$, the expected utility function $U(x;r)=rU_T(x)+(1-r)U_F(x)$ has a maximum at $x=r$.

Propriety can also be seen to follow from a continuity assumption on U_T and U_F and two constraints on one's rational method for assigning credences, along with the assumption that there *is* a rational method for assigning credences. We want our rational method for assigning credences to satisfy two plausible criteria. The first is 'precision': the method can potentially return any real-numbered credence value in the interval (0,1). After all, for any rational number, we can easily imagine a lottery situation where that rational number represents the obviously correct credence. The second is 'stable utility maximization': if the method yields some credence value, maximization of epistemic utilities based on that returned credence will not require one to assign some other credence.[7]

[5] For a discussion in the context of scoring rules, see, e.g., Don Fallis, "Attitudes toward Epistemic Risk and the Value of Experiments," *Studia Logica* 86 (2007): 215–246.

[6] Cf. the die example in Joyce, "Accuracy and Coherence," 283.

[7] An anonymous reader suggested that one might want convergence rather than stability. But then the rational method for assigning credences will be to choose the value that is being converged to, rather than the method a single iterative step. And once we choose the value that is being converged to, we will still want stability to apply.

Consider, then, a method m of assigning credences that satisfies these two constraints. Given stable utility maximization, $U(x,r)$ must have a maximum at $x=r$ for every r that m can return, and given precision, every rational-numbered value in (0,1) must be returnable. To show that propriety follows, we just need to extend this to the endpoints $r=0$ and $r=1$ as well as to irrational values of r. If U_T and U_F are continuous, then $U(x,r)$ is a continuous function of x, and a simple limiting case argument shows that $U(x,r)$ has a maximum at $x=r$ even if $r=0$ or $r=1$ or r is irrational.

Following ideas of Joyce[8] one can also argue for propriety at least in the case of some special p by using a special case of Lewis's Principal Principle.[9] Suppose I know for sure that a stochastic process now beginning has a chance r of resulting in outcome A and a chance $1-r$ of resulting in B instead. Let p be the proposition that A will results. Then by the Principal Principle I should assign credence r to p. But if the pair is not proper, then assigning credence r to p in cases like this is not what maximizes objectively expected epistemic utility. Hence, if our credence assignments in such cases are both to match the Principal Principle *and* maximize objectively expected epistemic utility, the utility pairs should be proper.

Standard examples (after transposing from the scoring rule context) of proper pairs are the Brier rule which in our setting will correspond to $U_T(r)=1/4-(1-r)^2$ and $U_F(r)=1/4-r^2$ and the logarithmic rule which in our setting will correspond to $U_T(r)=\log r+\log 2$ and $U_F(r)=\log(1-r)+\log 2$.

Now we have the following simple result, where g' is the derivative of the function g:

Theorem 1: *If U_T and U_F satisfy (b) and (d), then $U_T'(r)=(1-r^{-1})U_F'(r)$ for r in (0,1) and so $U_T(r)=U_T(1/2)+\int_{1/2}^{r}(1-u^{-1})U_F'(u)\,du$.*

[8] Joyce, "Accuracy and Coherence," 279. Alan Hájek ("Arguments for – or against – Probabilism," *British Journal for the Philosophy of Science* 59 (2008): 814) criticizes Joyce's use of the Principal Principle on the grounds that it is not clear that a whole probability assignment could correspond to objective chances, but as an anonymous reader has pointed out this criticism does not apply to the single-case argument I am about to give.

[9] David Lewis, "A Subjectivist's Guide to Objective Chance," in *Studies in Inductive Logic and Probability*, Volume II, ed. Richard C. Jeffrey (Berkeley: University of California Press, 1980), 263–293.

This follows simply from the fact that if $U(x;r)$ is maximized at $x=r$, then the derivative $\frac{dU(x;r)}{dx}$ must vanish at $x=r$, which derivative is equal to $rU_T'(x)+(1-r)U_F'(x)$, so that if this is zero at $x=r$, we must have $rU_T'(r)=(r-1)U_F'(r)$, from which the first result in Theorem 1 follows. The second conclusion in the Theorem follows immediately from the first. Note that the result holds for functions defined on all of [0,1], and not just [1/2,1], if one extends the differentiability and continuity assumptions.

Finally, let us set a neutral point to our epistemic utilities by supposing:

(e) $U_T(1/2)=U_F(1/2)=0$.

This embodies a substantive assumption that the value of credential equipoise does not depend on whether p is true or false (the zero-value is a mere convenience for our later discussion – what matters here is that $U_T(1/2)=U_F(1/2)$). One might perhaps question this in the case of some propositions p. Perhaps assigning credence 1/2 to a sceptical proposition, such as that I am a brain in a vat, is epistemically worse if the proposition is false than if it is true. This worry *may* involve a confusion between epistemic and non-epistemic utilities. Moreover, the badness of assigning credence 1/2 to a false sceptical proposition may be accounted for by the fact that this forces one to assign non-high credence to many other propositions, and we should not double count when aggregating the utilities over all the propositions one believes. In any case, assigning supposing $U_T(1/2)=U_F(1/2)$ would have to be the way to go if we wanted our utilities not to depend on the particular proposition.

Given (a) and (e), $U_T(r)>0$ and $U_F(r)<0$ for $r>1/2$.

We can now give the Theorem from which the results mentioned in the introduction follow. Suppose $r>1/2$. $U_T(r)$ measures how much, epistemically speaking, one loves having credence r when p is true, and $-U_F(r)$ measures how much, epistemically speaking, one hates having credence r when p is false. So we can define the hate-love ratio $HL(r)=\frac{-U_F(r)}{U_T(r)}$ that measures how much more one hates having credence r when p is false than one loves having credence r when p is true.

Theorem 2: *Suppose that U_T and U_F satisfy (a)–(e). Then $HL(r)\geq\frac{2r-1}{1-2r+\log 4+2\log r}$ for $r>1/2$.*

The proof of this theorem is given in the Appendix. In particular, we get that $HL(1) \geq 1/(\log 4 - 1)$ and that $HL(0.95) > 2.345$.

One might be interested to know whether there is any particular pair U_T and U_F that yields precisely the hate-love ratio on the right-hand-side of the inequality. The answer turns out to be affirmative. First, let $U_F(r) = 1/2 - r$ for r in $[1/2,1]$. Then define $U_T(r) = \int_{1/2}^{r} (1 - u^{-1}) U_F'(u)\, du$, as would have to be the case for propriety according to Theorem 1. Since $U_F'(u) = -1$, the integral is easy and yields $U_T(r) = \log r + \log 2 - r + 1/2$ for $r \geq 1/2$. Then, for symmetry, define $U_T(r) = U_F(1-r) = r - 1/2$ and $U_F(r) = U_T(1-r) = \log(1-r) + \log 2 + r - 1/2$ for $r < 1/2$.

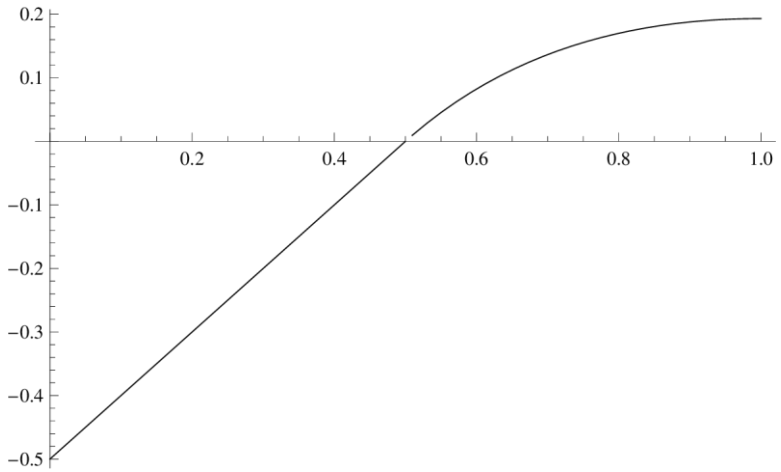

Figure 1: The function $U_T(r)$ that achieves equality in Theorem 2

It is easy to check that the right-hand side of the inequality in Theorem 2 then gives the exact hate-love ratio for this pair of functions. It is easy to see that U_F' is defined everywhere on $[0,1]$ (the point 1/2 is the only place where there could be a problem) and that it is decreasing. Hence U_F is concave on all of $[0,1]$. In the same way, we can check that U_T is concave.

The remaining thing to do is to check for propriety. Let $U(x;r) = rU_T(x) + (1-r)U_F(x)$. Then $\frac{dU(x;r)}{dx} = r(x^{-1} - 1) + r - 1$ for $x \geq 1/2$ and

$\frac{dU(x;r)}{dx}=r+(1-r)(1-(1-x)^{-1})$ for $x<1/2$. It is easy to check, considering the cases $r<1/2$ and $r\geq 1/2$ separately, that this derivative is positive for $x<r$ and negative for $x>r$, thereby showing that $U(x;r)$ has a strict maximum at $x=r$. Thus, this piecewise-defined proper pair of utilities achieves the smallest hate-love ratio possible. It might, thus, yield a scoring rule that will be of interest for further investigation.

3. Conclusions

The constraint that one's epistemic utility functions be proper, i.e., that it never make it irrational to stick to one's credences by the lights of these credences, together with a concavity constraint on the $r\geq 1/2$ part of the epistemic utility of believing in a falsehood, is sufficient to determine that these functions need to lopsidedly disfavor believing falsehoods over believing truth.

Plato famously argued in the *Republic* (587b–e) that the true king is 729 times happier than the tyrant. It may seem ridiculous that an exact number of such sort should appear in ethics. Yet, surprisingly, very natural assumptions do occasionally yield various numbers, as in our result that the epistemic disvalue of being certain and wrong is at least $1/(\log 4-1)$ times the epistemic value of being certain and right.[10]

Appendix: Proof of Theorem 2

First we need the following Lemma. It is basically a version of the FKG inequality, but I will need it under slightly different assumptions than those normally used in the FKG inequality, and hence I give a proof from scratch.[11]

Lemma 1: *Suppose f and g are non-negative functions on some interval $[a,b]$, with f monotone non-decreasing and g monotone non-increasing. Then:*

$$\int_a^b f(x)g(x)\,dx \leq \frac{1}{b-a}\int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx.$$

[10] I am grateful to Lara Buchak, Trent Dougherty, Jonathan Kvanvig and an anonymous reader for relevant discussion and/or comments.

[11] For the standard FKG inequality, see Geoffrey Grimmett, *Percolation* (New York: Springer, 1989), 34. In our setting we might not have the square-integrability assumption.

This lemma says that if f and g are monotone in opposite directions, we won't increase the integral of their product if we replace f with a constant function that has the same average value $(b-a)^{-1}\int_a^b f(x)\,dx$.

Proof of Lemma 1. For simplicity, assume that a=0 and b=1. The general case follows by rescaling. Use 1_A to denote the indicator function of the set A, i.e., a function that is 1 on A and 0 outside (with a contextually indicated domain). Suppose A is either $[0,\alpha]$ or $[0,\alpha)$ for some α in $(0,1]$. Then:

$$\begin{aligned}\int_0^1 1_A(x)f(x)\,dx &= \int_0^\alpha f(x)\,dx \qquad (1)\\ &\le \int_0^\alpha f(x/\alpha)\,dx=\alpha\int_0^1 f(u)\,du\\ &= \int_0^1 f(x)\,dx\cdot\int_0^1 1_A(x)\,dx,\end{aligned}$$

where the inequality followed from the fact that f is monotone non-decreasing and the subsequent equality followed by the change of variables $u=x/\alpha$. The overall inequality also trivially holds if α=0.

Now, let $g_\lambda = \{x\in[0,1] : g(x)>\lambda\}$ be the level set of g. Then for all x:

$$g(x)=\int_0^\infty 1_{g_\lambda}(y)\,dy. \qquad (2)$$

Observe also that g_λ is always an interval of the form $[0,\alpha]$ or $[0,\alpha)$ as g is non-increasing, and so (1) applies and yields:

$$\int_0^1 1_{g_\lambda}(x)f(x)\,dx\le\int_0^1 f(x)\,dx\cdot\int_0^1 1_{g_\lambda}(x)\,dx. \qquad (3)$$

Since one can reorder integrals of non-negative functions, we can use (2) twice and (3) once to conclude:

$$\int_0^1 f(x)g(x)\,dx = \int_0^1 f(x)\int_0^\infty 1_{g_\lambda}(x)\,d\lambda\,dx$$

$$= \int_0^\infty \int_0^1 1_{g_\lambda}(x) f(x)\, dx\, d\lambda$$

$$\leq \int_0^\infty \left(\int_0^1 f(x)\, dx \int_0^1 1_{g_\lambda}(x)\, dx \right) d\lambda$$

$$= \int_0^1 f(x)\, dx \cdot \int_0^1 \int_0^\infty 1_{g_\lambda}(x)\, d\lambda\, dx$$

$$= \int_0^1 f(x)\, dx \cdot \int_0^1 g(x)\, dx.$$

And in the special case where a=0 and b=1, that is the desired result. □

Proof of Theorem 2. For brevity, write $D_F(r)=-U_F(r)$ (this is the *disutility* of having credence r in the falsehood p). Then D_F is a non-decreasing monotone function, and it is convex on [1/2,1]. By Theorem 1 and since $U_T(1/2)=0$, we have:

$$U_T(r) = \int_{1/2}^r (u^{-1}-1) D_F'(u)\, du.$$

But u^{-1} is a monotone decreasing function while $D_F'(u)$ is monotone non-decreasing since D_F is convex as U_F is concave. By Lemma 1 and since $D_F(1/2)=0$ we have:

$$U_T(r) \leq \frac{1}{r-1/2} \int_{1/2}^r D_F'(u)\, du \cdot \int_{1/2}^r (u^{-1}-1)\, du$$

$$= \frac{1/2-r+\log 2+\log r}{r-1/2} \cdot \int_{1/2}^r D_F'(u)\, du$$

$$= \frac{1/2-r+\log 2+\log r}{r-1/2} \cdot D_F(r).$$

The fraction in front of $D_F(r)$ here must be positive since it is the integral of a function that is positive on [1/2,1). It follows that $D_F(r)/U_T(r) \geq \frac{r-1/2}{1/2-r+\log 2+\log r}$, which is equivalent to the desired result. □

NOT-EXACT-TRUTHS, PRAGMATIC ENCROACHMENT AND THE EPISTEMIC NORM OF PRACTICAL REASONING

Michael J. SHAFFER

ABSTRACT: Recently a number of variously motivated epistemologists have argued that knowledge is closely tied to practical matters. On the one hand, radical pragmatic encroachment is the view that facts about whether an agent has knowledge depend on practical factors and this is coupled to the view that there is an important connection between knowledge and action. On the other hand, one can argue for the less radical thesis only that there is an important connection between knowledge and practical reasoning. So, defenders of both of these views endorse the view that knowledge is the norm of practical reasoning. This thesis has recently come under heavy fire and a number of weaker proposals have been defended. In this paper counter-examples to the knowledge norm of reasoning will be presented and it will be argued that this view – and a number of related but weaker views – cannot be sustained in the face of these counter-examples. The paper concludes with a novel proposal concerning the norm of practical reasoning that is immune to the counter-examples introduced here.

KEYWORDS: pragmatic encroachment, practical reasoning, approximate truth, knowledge

1. Introduction

Recently a number of variously motivated epistemologists have argued that knowledge is closely tied to practical matters – let us refer to them here as the *encroachers* and to their view as *pragmatic encroachment*.[1] The most radical versions of this view are proposed as alternatives to epistemological views that are versions of intellectualism.[2] Intellectualists hold that knowledge does not depend in any way on practical factors and intellectualism is both deeply and commonly

[1] Of course various proponents of classical pragmatism such as William James, C.S. Peirce and F. P. Ramsey have also suggested related views previously. In particular, they variously suggested pragmatic views of belief where belief is intimately tied to action. However, these more historical views are not the topic of this paper and they will be ignored in what follows. See Michael Shaffer, "The Ramsey Principle and the Principle of Informational Equilibrium," *The Reasoner* 5 (2011): 37-39 for a related criticism of pragmatic theories of belief.

[2] The term 'intellectualism' originated with Earl Conee according to Jason Stanley, *Knowledge and Practical Interests* (Oxford: Oxford University Press, 2005), 6.

held by epistemologists. Nevertheless, those who defend radical forms of pragmatic encroachment reject intellectualism and hold that what counts as knowledge depends in some important sense on practical factors. This version of the view is then a serious challenge to a widespread and deep orthodoxy in epistemology and the sorts of cases that motivate radical encroachers are cases where it is supposed that what an agent is doing has significance with respect to what they know. This is primarily because of what is supposed be at stake in those cases. What radical encroachers claim is that by examining pairs of cases that differ only in terms of the stakes involved, we can see that such variation results in knowledge being present or absent.[3] Radical encroachers believe that the analysis of knowledge itself is infected by pragmatic concerns. Let us examine one such infamous pair of cases as presented by Jason Stanley to see this point that is so crucial to the radical encroacher's view:

> HANNAH AND SARAH 1: Hannah and her wife Sarah are driving home on a Friday afternoon. They plan to stop at the bank on the way home to deposit their paychecks. It is not important that they do so, as they have no impending bills. But as they drive past the bank, they notice that the lines inside are very long, as they often are on Friday afternoons. Realizing that it isn't very important that they paychecks are deposited right away, Hannah says, "I know the bank will be open tomorrow, since I was there just two weeks ago on Saturday morning. So we can deposit our paychecks tomorrow morning."[4]
>
> HANNAH AND SARAH 2: Hannah and her wife Sarah are driving home on a Friday afternoon. They plan to stop at the bank on the way home to deposit their paychecks. Since they have an impending bill coming due, and very little in their account, it is very important that they deposit their paychecks by Saturday. Hannah notes that she was at the bank two week before on a Saturday morning, and it was open. But, as Sarah points out, banks do change their hours. Hannah says, "I guess you're right. I do not know that the bank will be open tomorrow."[5]

Radical encroachers claim that Hannah knows that the bank will be open tomorrow in the first case, but not in the second, and that this is because of the difference in the practical interests of the agents in the two cases. So on this view, whether an agent knows is then not just a matter of non-practical factors and whether one knows depends on what interests one has.[6] More specifically, in low

[3] Kvanvig concurs about this analysis of the encroacher's strategy in this respect in Johnathan Kvanvig, "Against Pragmatic Encroachment," *Logos & Episteme* 2 (2011): 77-85.

[4] Stanley, *Knowledge and Practical Interests*, 3-4.

[5] Stanley, *Knowledge and Practical Interests*, 4.

[6] Jonathan Hawthorne, *Knowledge and Lotteries* (Oxford: Oxford University Press, 2004), Stanley, *Knowledge and Practical Interests*, Jonathan Hawthorne and Jason Stanley,

stakes situations knowledge is taken to be more prevalent than in higher stakes situations. As the pair of examples then shows, radical encroachers believe that two agents can be in epistemically similar situations qua their evidence, psychological situation and beliefs, while differing with respect to what they know because of the stakes involved and because of what they are doing. Such radical versions of pragmatic encroachment then also give rise to the additional view that knowledge is intimately connected to practical reasoning and action. So on this sort of view knowledge is closely related to action via the stakes dependence of knowledge itself and one ought only to act on (or deliberate using) what one knows.[7]

However one can defend the view that one ought only to act on (or deliberate using) what one knows without committing one's self to pragmatic encroachment. On the one hand, pragmatic encroachment is the view that facts about whether an agent has knowledge depend on practical factors and this is coupled to the view that there is an important connection between knowledge and action.[8] Typically this amounts to the claim that knowledge has intrinsically pragmatic content and that this needs to be reflected in the analysis or characterization of knowledge itself. On the other hand, one can argue there is an important connection between knowledge and practical reasoning in the sense that knowledge is the norm of practical reasoning without arguing that this needs to be reflected in the analysis or characterization of knowledge.[9] What is most important here is that the defenders of both of these views endorse the view that knowledge is the norm of practical reasoning. That is to say that all of these views involve accepting that claim that one should depend on a proposition is practical reasoning if and only if it is known. This thesis has recently come under heavy fire and a number of weaker proposals have been defended. In this paper counter-

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"Knowledge and Action," *The Journal of Philosophy* 105 (2008): 571-590, Timothy Williamson, *Knowledge and its Limits* (Oxford: Oxford University Press, 2000), Timothy Williamson, "Contextualism, Subject-Sensitive Invariantism and Knowledge of Knowledge," *The Philosophical Quarterly* 55 (2005): 213-235 and Jeremy Fantl and Matt McGrath, *Knowledge in an Uncertain World* (Oxford: Oxford University Press, 2009) are all defenses of versions of pragmatic encroachment.

[7] See for example Hawthorne, *Knowledge and Lotteries*, 30 and Stanley, *Knowledge and Practical Interests*, 8-12.

[8] John Hawthorne, Jason Stanley, Jeremy Fantl and Matthew McGrath are encroachers of this sort.

[9] Timothy Williamson appears to be an encroacher of this sort, particularly because he believes that knowledge is not analyzable. This is of course expressed by his infamous endorsement of the E = K thesis. See Williamson, *Knowledge and its Limits* and Patrick Greenough and Duncan Pritchard, *Williamson on Knowledge* (Oxford: Oxford University Press: 2009).

examples to the knowledge norm of reasoning will be presented and it will be argued that this view – and a number of related but weaker views – cannot be sustained in the face of these counter-examples. The paper concludes with a novel proposal concerning the norm of practical reasoning that is immune to the counter-examples introduced here.

2. Pragmatic Encroachment and the Knowledge Norm of Practical Reasoning

There are a number of specific versions of the knowledge norm of practical reasoning, but the versions defended by Timothy Williamson and by John Hawthorne and Jason Stanley are perhaps the most well-known and influential. Hawthorne and Stanley are proponents of the typical form of strong pragmatic encroachment discussed above, whereas Williamson is a proponent only of the knowledge norm of practical reasoning. So they all share in common the view that knowledge is the norm of practical reasoning. In adopting this view Williamson and Hawthorne and Stanley have independently endorsed what amounts to the following thesis:[10]

> (KNPR) Ksp ≡ it is rational for S to employ p (appropriately) in S's practical reasoning.[11]

To support this attribution one need only look at the following rather straightforward claims to this effect. Williamson claims that, "…one knows q iff q is an appropriate premise for one's practical reasoning."[12] Hawthorne and Stanley similarly claim that, "Where one's choice is p-dependent, it is appropriate to treat the proposition that p as a reason for acting iff you know that p."[13] So, all three are all encroachers in this sense and the attribution of KNPR to them appears to be correct. Of course, this means that its being rational for S to employ p (appropriately) in S's practical reasoning is both a necessary and sufficient condition for S's knowing that p. So, we can represent the necessary condition component of the view easily and conveniently as follows:

> (KNPR-N) It is rational for S to employ p (appropriately) in S's practical reasoning → Ksp.

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[10] See Williamson, "Contextualism, Subject-Sensitive Invariantism," Hawthorne and Stanley, "Knowledge and Action," Williamson, *Knowledge and its Limits*, Hawthorne, *Knowledge and Lotteries*, and Stanley, *Knowledge and Practical Interests*.

[11] Lackey refers to this general view as the knowledge norm of practical reasoning (KNPR) in Jennifer Lackey, "Acting on Knowledge," *Philosophical Perspectives* 24 (2010): 361-382. This usage will be followed here.

[12] Williamson, "Contextualism, Subject-Sensitive Invariantism," 231.

[13] Hawthorne and Stanley, "Knowledge and Action," 578.

Of course, one might immediately take issue with KNPR (and hence with KNPR-N) by noting that what counts as 'rational' with respect to practical reasoning and what counts as 'appropriate' use(s) of p are controversial matters to say the least. However, here KNPR will be challenged by presenting a perfectly clear and potent counter-example to the claim that S's knowing that p is a necessary condition for it to be rational for S to employ p (appropriately) in S's practical reasoning – that is to say, KNPR will be challenged by producing a clear counter-example to KNPR-N. What will make this even more telling as a counter-example is that the case involved does not solely turn on the degree to which the agent is justified in believing the relevant proposition(s) in the case in question and the case has little or nothing to do with the stakes involved. But, first we need to consider two clarificatory points about the pragmatic encroachers' views with respect to KNPR.

First, it is important to note the qualification concerning p-dependence that Hawthorne and Stanley impose on their version of KNPR.[14] This condition is imposed in order to allow them to deal with certain problematic counter-examples to the knowledge norm of practical reasoning involving acting on the basis of irrelevant propositions. For Hawthorne and Stanley a choice between options $\{o_1, o_2,....,o_n\}$ is *p-dependent* if and only if the most preferable of $\{o_1, o_2,....,o_n\}$ given proposition p is not the same as the most preferable option given the proposition $\neg p$.[15] Most defenders of the knowledge norm have then followed Hawthorne and Stanley in imposing this qualification on KNPR in order to deal with those sorts of counter-examples, and so it has become a canonical element of the general view.

Second, let us say a little more about KNPR and the relevant concept of appropriateness that it assumes. As Jennifer Lackey has recently pointed out, what the defenders of KNPR are specifically interested in is *epistemic* appropriateness, as opposed to moral appropriateness, societal appropriateness, etc.[16] So, she characterizes the knowledge norm view generally as follows:

> It is epistemically appropriate for one to use the proposition that p in practical reasoning if and only if one knows that p.[17]

So she essentially attributes KNPR, and thereby KNPR-N, to the encroachers as well, but makes an important additional clarification of their view. Lackey's characterization is then particularly instructive in this respect because it specifies the particular sense of appropriateness assumed in KNPR and KNPR-N (i.e.

[14] Hawthorne and Stanley, "Knowledge and Action."
[15] See Hawthorne and Stanley, "Knowledge and Action," 4.
[16] Lackey, "Acting on Knowledge."
[17] Lackey, "Acting on Knowledge," 1.

epistemic appropriateness). So, we can make this explicit as follows. Where the choice if p-dependent,

> (KNPR') Ksp ≡ it is *epistemically* rational for S to employ p (appropriately) in S's practical reasoning.

And we can do the same thing with PE-N as follows. Where the choice is p-dependent,

> (KNPR-N') It is *epistemically* rational for S to employ p (appropriately) in S's practical reasoning → Ksp.

This too has become a component of the canonical general version of KNPR.

3. Two Counterexamples to KNPR

Having made this point, let us now turn to developing the promised counter-example.

> ROBIN 1: suppose that Robin is an independently wealthy advanced physics student studying relativistic mechanics at M.I.T. in 2009 and who is totally ignorant about archery. Nevertheless he believes that the laws of Newtonian mechanics are false and he knows that he believes this. Suppose then that Robin has been offered a fairly standard sort of performance wager. The terms of the wager are as follows. Robin will be given a bow and arrow, although Robin has never previously used this bow or one of its type. Nevertheless, he is aware of the strength of the bow and is assured that it is perfectly functional, that the arrow is perfectly normal and that anyone can use it effectively without training. He is also allowed to train as much as he likes. Robin is asked to put up $50, and provided he can shoot an arrow beyond a marker set at 30 yards he will win $100. If he fails to do so, then he loses the $50 he put up. So, Robin must use his practical reason to determine whether he should accept the wager or not. Initially, given his ignorance of archery, he has no idea whether the bow is capable of shooting an arrow beyond the marker. But recall that Robin is a physics student and so let us suppose that Robin quickly remembers Newton's laws of force and motion and so easily calculates that given the strength of the bow and a reasonable angle of trajectory, the arrow will travel at least 100 yards. So, Robin completes his practical deliberation, accepts the wager and proceeds to win, thus doubling his stake.

Now, it should be patently clear that Robin's behavior is *epistemically* rational given virtually any standard of epistemic rationality. Robin deliberates about accepting the wager in a perfectly rational manner on the basis of propositions that it is perfectly rational for him to employ and draws the perfectly reasonable conclusion that he should accept the wager. He is also fact successful in his acting on the basis of his reasoning to this effect. But, in the course of Robin's

practical reasoning, he depends essentially on a number of propositions that are constitutive of Newtonian mechanics (i.e. Newton's laws of force and motion). Of course, we (and *ex hypothesi* Robin) know that these propositions are only approximately true.[18] Newton's laws only hold (approximately) for cases where the velocities involved do not approach the speed of light. In reality, we (and *ex hypothesi* Robin) are aware that, properly speaking, relativistic mechanics describes the motion of the arrow and its possible trajectories. However, in the circumstances Robin finds himself in, the equations of Newtonian mechanics are sufficiently close to the relativistic case so that Robin can – in what looks like the correct epistemic sense – rationally depend on them, even though he cannot possibly know them. He cannot possibly know them because, strictly speaking, approximately true claims are false and knowledge is widely taken to be factive.[19] This makes the counter-example especially potent because the encroachers cannot reasonably claim that Robin really does know the relevant proposition(s) p in this case, and this is the case because the example does not turn at all on Robin's degree of justification for the propositions of Newtonian mechanics, for he *knows* that those propositions are false. So, we have a p-dependent choice and it seems to be obviously *epistemically* rational for S to employ p (appropriately) in S's practical reasoning but where it is clear that $\neg K_S p$. So, KNPR-N' appears to be false, and thereby KNPR' is impugned as well.

Now, one obvious retort is that while Robin cannot know the laws of Newtonian mechanics because they are false but approximately true, he can certainly know *that they are approximately true*. In point of fact, *ex hypohesi* this belief is adequately justified in this case. One might then submit that it is Robin's knowledge of the approximate truth of those propositions that is the basis of his

[18] They can be regarded as approximately true in the standard sense (see Graham Oddie, "Truthlikeness," *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, ed. Edward N. Zalta, URL= <http://plato.stanford.edu/archives/fall2008/entries/truthlikeness/>), partially true (see Elijah Millgram, *Hard Truths* (London: Wiley-Blackwell, 2009)), false but true enough (see Catherine Elgin, "True Enough," *Philosophical Issues* 14 (2004): 113-131), or inexactly true (see Paul Teller, "Twilight of the Perfect Model," *Erkenntnis* 55 (2001): 393-415 and Paul Teller, "The Finewright Theory," in *Nancy Cartwright's Philosophy of Science*, eds. Stephan Hartmann, Carl Hoefer, and Luc Bovens (London: Routledge, 2008), 91-116). The counter-examples here will work – with only very minor modification – for any of these alternative accounts of useful claims that are 'true-but-not-exactly-true.'

[19] See Risto Hilpinen, "Approximate Truth and Truthlikeness," in *Formal Methods in the Methodology of the Empirical Sciences*, eds. Marian Przelecki, Klemens Szaniawski, and Ryszard Wojcicki (Dordrecht: Reidel, 1976), 19-42, Theo Kuipers, *What is Closer-to-the-truth?* (Amsterdam: Rodopi, 1978), Graham Oddie, *Likeness to Truth* (Dordrecht: Reidel, 1986) and Oddie, "Truthlikeness."

deliberation about whether to accept the wager. But, of course we can simply modify the story that introduces the counter-example slightly to avoid this riposte. Let us alter our story then as follows.

> ROBIN 2: let us now suppose that Robin is an independently wealthy physics student studying Newtonian mechanics in 1795, that he believes that Newton's laws of force and motion are true, that this belief is adequately justified and that he is totally ignorant about archery. So he believes that the laws of Newtonian mechanics are true and he knows that he believes this. Let us then suppose that Robin is offered the same performance wager as in the first case and that the terms of the wager are the same. So, again Robin must use his practical reason to determine whether he should accept the wager or not. Now recall that since Robin is a physics student we can imagine that he quickly remembers Newton's laws of force and motion and so easily calculates that given the strength of the bow and a reasonable angle of trajectory, the arrow will travel at least 100 yards. So, Robin completes his practical deliberation, accepts the wager and proceeds to win, thus doubling his stake.

In this case, it still seems epistemically appropriate for Robin to employ the propositions that constitute Newton's mechanics in his deliberations and his choice is p-dependent for the same reason that apply in the case of ROBIN 1, but he does not know – or even believe – that they are approximately true in this case (even though they are in point of fact only approximately true), and, of course, he does not know that they are true. So, the counter-example is easily saved from this sort of response and the encroacher's view is in serious trouble unless they (1) claim that Robin *really* does know the propositions of Newtonian mechanics in the second case, or (2) they claim that Robin somehow *really* does know that the propositions of Newtonian mechanics are approximately true or close enough to true in the second case, or (3) they adopt the view that in the second case Robin is not acting in an epistemically appropriate manner. But none of these options seems reasonable at all and so the encroacher's view is in serious jeopardy. Strategy (1) fails straightforwardly because knowledge is factive. Strategy (2) fails (*ex hypothesi*) because Robin does not believe, let alone know, that the propositions of Newtonian mechanics are approximately true. Finally, strategy (3) would ultimately require adopting the totally implausible view that it is never epistemically rational to base one's practical reasoning on approximately true premises that one is adequately justified in believing. It is worth noting then that cases like the Robin cases are utterly pedestrian and so the jeopardy is both serious and widespread. It is often perfectly rational to base one's practical reasoning on propositions that one has adequate justification for but which are only approximately true, even when we do not know that they are only approximately

true. In fact, we typically do this because these sorts of propositions have great practical value. This happens with great regularity in both everyday reasoning and in sciences and we shall return to this topic shortly.

4. Weakened forms of KNPR

It is tempting however to believe that there are weaker forms of KNPR that avoid the counter-examples proposed here and two such responses have recently been proposed. The first alternative suggestion that seems relevant here is related to a position on the matter that has been endorsed recently by Ram Neta.[20] Neta also argues against KNPR and then argues that all of the examples offered by Hawthorne and Stanley in support of KNPR can be explained by a weaker version of that thesis.[21] His discussion suggests that the following modification of KNPR' might be used to avoid the negative conclusion about the encroacher's view based on the Robin cases. Where the choice involved is p-dependent,

> (JBKNPR) JBsKsp ≡ it is epistemically rational for S to employ p (appropriately) in S's practical reasoning.

Here JBsKsp just means that S justifiably believes that she knows that p. We can then derive the corresponding weaker version of KNPR-N' as follows: where the choice is p-dependent,

> (JBKNPR-N) It is epistemically rational for S to employ p (appropriately) in S's practical reasoning → JBsKsp.

Neta's proposal however fails straightforwardly as it cannot accommodate ROBIN 1. In the first Robin case Robin's reasoning and subsequent action is epistemically rational, but he is not justified in believing that he knows that the laws of Newtonian mechanics are true. This is simply because he knows that they are false. He is however justified in believing that those laws are approximately true or close enough to true for his purposes and so this is suggestive of how one might modify JBKNPR in order to get the correct results in the Robin cases. What needs to be worked into JBKNPR in order to avoid the threat of ROBIN 1 is the requirement that one's beliefs used in practical reasoning need only to be known to be approximately true, and that they need not be known simpiciter. If we leave JBKNPR as it is stated however, ROBIN 1 refutes Neta's weakened version of KNPR. Neta's proposal appears however to do better in the case of ROBIN 2. In ROBIN 2 Robin is justified in believing that he knows that the laws of Newtonian

[20] See Ram Neta, "Treating Something as a Reason for Knowledge," *Nous* 43 (2009): 684-699.

[21] See Neta, "Treating Something" and Hawthorne and Stanley, "Knowledge and Action."

mechanics are true because he has a great deal of evidence supporting the truth of that theory. This is not really the case however because Neta's proposal then faces a damning dilemma with respect to ROBIN 1 and ROBIN 2 *taken as a pair of counterexamples*.

On the one hand, JBKNPR can be maintained as is (i.e. without weakening it to require only that one justifiably believes that one knows that the propositions being used in one's practical deliberations are approximately true). But, then JBKNPR fails due to ROBIN 1. On the other hand, one could weaken JBKNPR and require only that one justifiably believes that one knows that the propositions being used in one's practical deliberations are approximately true. But, then JBKNPR fails due to ROBIN 2. One would have to maintain in ROBIN 2 that while Robin does not know that the laws of Newtonian mechanics are true and he does not know that they are approximately true, he is acting in an epistemically appropriate manner because he is justified in believing that he knows that the laws of Newtonian mechanics are approximately true. But this won't do at all. If JBKNPR is to save the encroacher's view from the threat posed by ROBIN 2 it would have to be the case that (a) Robin is justified in believing that he believes that the laws of Newtonian mechanics are approximately true, (b) he would have to be justified in believing that it is true that the laws of Newtonian mechanics are approximately true and (c) he would have to be justified in believing that he is justified in believing that the laws of Newtonian mechanics are approximately true in 1795 and as the case is described. These three claims are true (respectively) because knowledge presupposes belief, is factive and requires adequate justification.

It is clear however that the first of these three claims is not true in ROBIN 2. Robin believes falsely in that case that the laws of Newtonian mechanics are true and he knows that he believes that they are true. Given his evidence he is clearly not justified in believing that he believes that those laws are approximately true and he knows that he does not believe that the laws of Newtonian mechanics are approximately true. It is also not entirely obvious that Robin meets the second condition. Given his evidence in 1795 it is not at all obvious that he is justified in believing that the laws of Newtonian mechanics are approximately true, particularly if we are treating justification in terms of internalism. Finally, it also not entirely obvious that the third claim is true in the second Robin case. Since Robin does not believe that the laws of Newtonian mechanics are approximately true in that scenario it would be exceedingly strange to say that he is justified in believing that he is justified in believing that the laws of Newtonian mechanics are approximately true. However, whatever one says about (b) and (c), the fact that (a)

is not met in ROBIN 2 while it is epistemically rational for Robin to use the claim that Newton's laws are approximately true in his deliberations in that scenario shows that Neta's weakened view of KNPR is in serious trouble. This also indicates more generally that the defenders of KNPR are in trouble because we have plausible counter-examples with respect to both KNPR-N' and with respect to JBKNPR-N, and so with respect to KNPR' and with respect to JBKNPR. So it appears to be the case that it can be epistemically rational to act on a false but approximately true proposition, even when one is not justified in believing that one knows it is true that such a proposition is approximately true.[22]

A second recently proposed alternative that is relevant here has been endorsed by Clayton Littlejohn. Littlejohn, like Neta, argues against KNPR and claims that none of the examples offered Hawthorne and Stanley support KNPR.[23] What is more interesting is that he endorses what amounts to the following principle to replace both KNPR and JBKNPR.[24] Where the choice is p-dependent,

> (JBTNPR) (JBsp) and (p is true) ≡ it is epistemically rational for S to employ p (appropriately) in S's practical reasoning.

We can then derive the corresponding weaker version of KNPR-N. Where the choice is p-dependent,

> (JBTNPR-N) It is epistemically rational for S to employ p (appropriately) in S's practical reasoning → (JBsp) and (p is true).

Littlejohn claims that this principle is the weakest principle that allows us to say of an agent both that he is concerned about the accuracy of his beliefs and that he is concerned with his concern for the accuracy of his beliefs. He then explains that,

> ...it makes sense to say that if p misrepresents how things are or the subject arrives at the belief that p in a way that only someone insufficiently concerned with the truth could have, it follows that the subject's belief that p is not proper and is not the proper basis for further deliberation.[25]

But, even this principle is too strong, and ROBIN 1 and ROBIN 2 illustrate this clearly. Robin's actions in both cases are epistemically rational, but the propositions used by Robin in his practical deliberations certainly do not meet Littlejohn's factive condition that p be true. Again, as we have already seen, it

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[22] See Elgin, "True Enough" for discussion of the utility of false but 'true enough' beliefs.

[23] See Clayton Littlejohn, "Must We Act Only on What We Know," *The Journal of Philosophy* 106 (2009): 463-473 and Hawthorne and Stanley, "Knowledge and Action."

[24] Littlejohn refers to this principle as RJTBP in Littlejohn, "Must We Act."

[25] Littlejohn, "Must We Act," 473.

appears to be the case that it can be epistemically rational to act on a false but approximately true proposition. So the basis of practical reasoning has to something weaker yet and the norm of practical reasoning does not appear to be knowledge *simpliciter*. It is then also worth noting that attempts to weaken KNPR like Neta's and Littlejohn's result in views that are no longer about knowledge. They are merely about justified belief. As a result, pragmatic encroachers who adopt the weakening strategy threaten both to undermine their stated motives and to make their view uninteresting – at least if they continue to maintain that their view is a revisionary theory of knowledge.[26]

5. Practical Reasoning, Rationality and Not-Exact-Truths

Taking the lessons of the previous two sections to heart, a plausible candidate for a principle concerning the epistemic conditions on practical reasoning might then be something like the following one. Where the choice is p-dependent,

> (JBATNPR) (It is at least the case that JBsp is approximately true) and (p is at least approximately true) ≡ it is epistemically rational for S to employ p (appropriately) in S's practical reasoning.

It is important to notice that the justified belief component of the left hand side of the bi-conditional is qualified by an 'at least' qualification with its scope outside the doxastic operator. This is intentionally designed to capture the idea that the norm of practical reasoning involves at least S being justified in her belief that p is approximately true. This is then compatible with S's being justified in her belief that p is strictly true as well her being justified in her belief that p is only approximately true. We cannot just substitute (JBsp is at least approximately true) for (JBsp or JBsp is approximately true) without running into problems in the second Robin case. So that particular qualification is crucial. If we insisted on the condition with the scope as follows: (JBsp is at least approximately true), then ROBIN 1 would not be too problematic. In ROBIN 1 this condition is met because Robin is justified in believing that the laws of Newtonian mechanics are approximately true and it is not unreasonable to suppose that Robin would also have the belief that those laws are then at least approximately true. In ROBIN 2 this condition would not be met because Robin is justified in believing that the laws of Newtonian mechanics are true, but it is not reasonable to suppose that he has the belief that they are approximately true or even at least approximately true.[27] Additionally, adopting that condition might give rise to the appearance that

[26] This point about weakening KNPR was suggested by a very helpful referee.

[27] This worry was suggested by a referee.

meeting the condition would be inferential and that would be problematic were one to deny the rationality of standard closure principles. So it looks like the condition with the qualifier outside the scope of the doxastic operator is the right way to go in defining the norm of practical reasoning. In the other conjunct in the left hand side of the bi-conditional p's being at least approximately true signifies that p is true *or* that p is approximately true.

So this much weaker principle captures a much more reasonable sense of the epistemic conditions on practical reasoning and it has two important virtues. First and foremost, it gets us the correct result in both the ROBIN 1 and ROBIN 2 cases. In ROBIN 1 the M.I.T. Student is at least justified in his belief that the laws of Newtonian mechanics are approximately true because, *ex hypothesi*, he knows the conditions under which the approximations involved are appropriate, and those laws are in fact approximately true. In ROBIN 2 the 1795 counterpart of our contemporary M.I.T. student is justified in believing that the laws of Newtonian mechanics are true *simpliciter* so he too is at least justified in his belief that the laws of Newtonian mechanics are approximately true. Moreover, they are in fact approximately true. So JBATNPR appear to get things right with respect to the norm of practical reasoning, but it involves significant weakening of KNPR and even of its weaker cousins.

Second, this weak principle of the epistemic conditions on practical reasoning respects what a number of variously motivated philosophers have convincingly argued about epistemic rationality and inexact truth to a much greater extent than do any of the other proposals. This is interesting because the parties to the debate about pragmatic encroachment and the defenders of the knowledge norm of practical reasoning have by and large simply assumed some implicit philosophical or folk theory of rationality in the discussion of these ideas that ignores the practical rationality of inexact, partial or approximate truths. Serious discussion of the substance of rationality itself is conspicuously absent in all of Williamson's, Hawthorne and Stanley's, Neta's and Littlejohn's papers and so this is not really a surprise.[28] But, this lacuna is problematic here because what a number of other philosophers have recently and compellingly argued is that rational thinking and acting involves the use of approximations, idealizations and/or inexact truths.[29] That we are less than perfectly rational is, of course, not at

[28] See Williamson, "Contextualism, Subject-Sensitive Invariantism," Hawthorne and Stanley, "Knowledge and Action," Neta, "Treating Something," and Littlejohn, "Must We Act."

[29] See Catherine Elgin, *Considered Judgment* (Princeton: Princeton University Press, 1996), Elgin, "True Enough," Nancy Cartwright, *How the Laws of Physics Lie* (Oxford: Oxford University Press, 1983), Millgram, *Hard Truths*, Teller, "Twilight of the Perfect Model" and

all a new recognition and the debates between the various defenders of the heuristics and biases tradition, the ecological rationality model and more traditional views attests to this.[30] We do not need to go into the details of these debates here, but what they strongly suggest is that we sometimes base both practical and theoretical reasoning on propositions that are not-exactly-true and that we can be efficient problem solvers and deliberators even though we do not reason in maximally accurate ways on the basis of exact truths.[31] We often trade degrees of accuracy with respect to truth for things like efficiency, ease of use and generality – just as Robin does in ROBIN 1 and ROBIN 2 – but without compromising rationality or success. There is nothing irrational about employing approximate, partial or inexact truths in our practical reasoning and JBATNPR reflects this whereas the stronger alternatives discussed above simply do not do so. In that respect JBATNPR is more realistic.

6. Objections, Responses and Implications

Let us then turn to the consideration of some possible objections that the pragmatic encroacher's might raise about the cases and to some responses they might give to the results derived from the cases. The first two objections involve claims that the cases are misdescribed in some important way and that the factivity condition is met in both Robin cases. The first such objection involves looking at some worries about the relationship between Newtonian mechanics and relativistic mechanics. The second such objection, involves some potential worries about the relationship between truth and approximate truth. Two more radical responses to the results presented here involve conceding the descriptive correctness of the cases, and rejecting the conclusions drawn on the basis of those cases nonetheless. The first such response involves the rejection of factivity and the other response involves the adoption of the safety condition on knowledge. Ultimately it will be shown here that all of these objections and responses are inadequate, but they are worth looking at nonetheless.

"The Finewright Theory," Mark Wilson, *Wandering Significance* (Oxford: Oxford University Press, 2006), and William Wimsatt, *Re-engineering Philosophy for Limited Beings: Piecewise Approximations to Reality* (Cambridge: Harvard University Press, 2007).

[30] See, for example, Renée Elio, ed., *Common Sense, Reasoning and Rationality* (Oxford: Oxfrod University Press, 2002), Massimo Piattei-Palmarini, *Inevitable Illusions* (New York: Wiley, 1994), Gerd Gigerenzer, *Adaptive Thinking* (Oxford: Oxford University Press, 2000), Michael Shaffer, "Decision Theory, Intelligent Planning and Counterfactuals," *Minds and Machines* 19 (2009): 61-92, and Michael Shaffer, *Counterfactuals and Scientific Realism* (New York: Palgrave MacMillan, 2012).

[31] See Shaffer, "Decision Theory, Intelligent Planning."

So, one way to challenge the results here would be to challenge the acceptability of the Robin cases in terms of the manner in which the relationship between Newtonian mechanics and relativity theory is understood in those cases. In the first Robin case it is assumed that Newtonian mechanics is false and that relativity theory is true. So the former cannot be known and the latter can be known. But, some philosophers of science do not accept these claims. Some philosophers of science argue that theories that have been superseded by better theories that capture them as restricted cases are not false.[32] So it would not follow that if relativity theory is true, then Newtonian mechanics is false. If this view were granted, then in the first Robin case Robin could know Newtonian mechanics because it is not false. The factivity condition on knowledge would be met and KNPR would be immunized against the Robin counter-examples. Robin would then have knowledge if he has the relevant beliefs about Newtonian mechanics and if those beliefs were justified. Other philosophers of science argue that *no* theories are true.[33] If this is the case, then in the first Robin case Robin would not know that relativistic mechanics is true and so it would not be rational for him to base his practical reasoning on that theory.

The problem with the first component of this objection is that it faces a damning dilemma. Newtonian mechanics can be understood as Newton proposed it or as a special case of relativistic mechanics – as it is understood contemporarily. As is well-known Newton proposed his theory unrestrictedly (i.e. it was claimed to hold at all velocities and for all masses). So if Newtonian mechanics is understood as Newton understood it (and as Robin would understand it in 1795), then it is false. Its observable implications have been found to be false and so it has been definitively falsified. Robin of course probably does not believe Newtonian mechanics in this sense in ROBIN 1, as he is aware of the relationship between the two theories of mechanics. So this appeal gets ROBIN 1 right and Robin could know Newtonian mechanics. If we understand Newtonian mechanics as having a restricted scope (i.e. as a special case of relativity theory) then it is true. But, that is *not* what Robin believes in ROBIN 2. We cannot non-anachronistically say that Newton's theory is a true special case of relativistic mechanics and that 1795 Robin believes *that*. So this suggestion cannot get the correct result in ROBIN 2.[34]

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[32] See Fritz Rorhlich and Larry Hardin, "Established Theories," *Philosophy of Science* 50 (1983): 603-617. This is of course not the orthodox view of the matter.

[33] Cartwright, *How the Laws of Physics Lie*.

[34] This response is the same response that one can give to versions of the correspondence principle famously defended by Bohr, Poincaré, and others. See Michael Shaffer, "Idealization, Counterfactuals and the Correspondence Principle," in *The Courage of Doing Philosophy: Essays Dedicated to Leszek Nowak*, eds. Jerzy Brzeziński, Andrzej Andrzej, and Theo A. F.

Thus, the pair of counter-examples would still refute KNPR. The problem with the second component of this objection is that it is not at all clear that *all* theories are false and that relativity theory cannot be known or rationally used in practical deliberations. It may well turn out that relativity theory itself *is false but approximately true*, and so it would be rational for Robin to base his reasoning on those propositions even if that theory is not strictly true. This was the upshot of JBATNPR.

Finally, if one is not entirely convinced by these responses to this objection, we can simply construct a different counterexample that does not involve theories at all. As a result, the objection is rendered moot. Consider the following case:

> ROBIN 3: suppose that Robin is an independently wealthy carpenter in 2009 and who is totally ignorant about archery. Suppose then that Robin has been offered a fairly standard sort of performance wager. The terms of the wager are as follows. Robin will be given a bow and arrow, although Robin has never previously used this bow or one of its type. Nevertheless, he is aware of the strength of the bow and is assured that it is perfectly functional, that the arrow is perfectly normal and that anyone can use it effectively without much training. He is allowed to train as much as he likes, however. Robin is asked to put up $50, and provided he can shoot an arrow beyond a marker set at 30 yards he will win $100. If he fails to do so, then he loses the $50 he put up. So, Robin must use his practical reason to determine whether he should accept the wager or not. Initially, given his ignorance of archery, he has no idea whether the bow is capable of shooting an arrow beyond the marker. So Robin takes 10 practice shots and uses his handy tape measure to determine to the closest tenth of a foot that arrows landed approximately 105.5 yards., 103.6 yards, 106.8 yards, 101.7 yards, 107.3 yards, 102.3 yards, 104.1 yards, 103.2 yards, 106.5 yards, and 103.3 yards away. So, on the basis of what – being a good carpenter – he knows to be only approximate measurements he concludes that an arrow fired from the bow will travel at least 100 yards. So, Robin completes his practical deliberation, accepts the wager and proceeds to win, thus doubling his stake.

So Robin is epistemically rational in his practical reasoning, but does not know the distances that the arrows really travelled. His beliefs about those distances are all only approximately true and so do not constitute knowledge and he knows this, but this does not preclude him from being rational in using them in his practical reasoning. More importantly, none of this depends on his beliefs about any scientific theory at all and so this objection can easily be circumvented in this manner. Moreover, making the analogous move that gave rise to ROBIN 2

Kuipers (Amsterdam: Rodopi, 2007), 179-204 and Michael Shaffer, "Re-formulating the Correspondence Principle: Problems and Prospects," *Polish Journal of Philosophy* 2 (2008): 99-115 for criticism of that view of inter-theoretical relations.

from ROBIN 1 and claiming that Robin knows that the distances the test arrows were fired are approximately true will not work here either because we can slightly alter the story as follows:

> ROBIN 4: suppose that Robin is an independently wealthy lounge singer in 2009 and who is totally ignorant about archery. Suppose then that Robin has been offered a fairly standard sort of performance wager. The terms of the wager are as follows. Robin will be given a bow and arrow, although Robin has never previously used this bow or one of its type. Nevertheless, he is aware of the strength of the bow and is assured that it is perfectly functional, that the arrow is perfectly normal and that anyone can use it effectively without much training. He is allowed to train as much as he likes, however. Robin is asked to put up $50, and provided he can shoot an arrow beyond a marker set at 30 yards he will win $100. If he fails to do so, then he loses the $50 he put up. So, Robin must use his practical reason to determine whether he should accept the wager or not. Initially, given his ignorance of archery, he has no idea whether the bow is capable of shooting an arrow beyond the marker. So Robin takes 10 practice shots and uses his handy tape measure to determine to the closest tenth of a foot that arrows landed 105.5 yards., 103.6 yards, 106.8 yards, 101.7 yards, 107.3 yards, 102.3 yards, 104.1 yards, 103.2 yards, 106.5 yards, and 103.3 yards away. So, on the basis of what he takes to be accurate measurements he concludes that an arrow fired from the bow will travel at least 100 yards. So, Robin completes his practical deliberation, accepts the wager and proceeds to win, thus doubling his stake.

Here Robin is again perfectly rational in his practical reasoning, but he does not even believe that the measurements are approximately true and so cannot know that to be the case. So this objection fails, whatever one might say about the relationship between Newtonian and relativistic mechanics.

A second – and closely related – way to challenge the Robin cases would be to challenge the assumption that if a proposition if approximately true, then it is false. Were one to adopt this view, then one could maintain that some approximately true theories are also true. If this were true of Newtonian mechanics, then that theory would be true as well as approximately true. So, in the first Robin case Robin could meet the factivity condition on knowledge and would know Newtonian mechanics, thus immunizing KNPR against that counterexample. As in the case of the first objection, Robin would then have knowledge in the first case because he has the relevant beliefs, they are justified and they are true. The problem with this view is that all extant theories of approximate truth are explicitly based on the claim that all approximately true propositions are strictly false, although there are many falsehoods that are not

approximately true.[35] So, if this strategy were to be pursued, we would be owed an account of approximate truth that does not incorporate this feature. But, it is difficult to see why one might want such a theory. The extant theories of approximate truth, truthlikeness and verisimilitude were developed specifically to make the distinction between not-true propositions that are just false and not-true propositions that are approximately true.[36] More importantly, this maneuver does nothing to immunize KNPR against ROBIN 2. Even if one could make sense of this idea in ROBIN 2 Robin does not believe that Newtonian mechanics is approximately true and so cannot reasonably be taken to believe that the theory is approximately true and true.

Encroachers however might just grant that the counterexamples are adequate with respect to the theories involved and with respect to the relationship between the concepts of truth and approximate truth, and simply attempt to dodge the criticism by arguing that in both cases Robin does know. The first way to do this involves the recognition that in both Robin cases the stakes are low. As we saw in section 1, encroachers believe that in low stakes situations knowledge is more prevalent than in high stakes situations. So encroachers can potentially respond to the Robin cases by arguing that in both cases Robin does know Newtonian mechanics even though the propositions that constitute that theory are false but approximately true. What an encroacher might then say is that in low stakes situations approximate truth is sufficient for knowledge and that these cases involve low stakes. This then amounts to the concession of the factivity condition on knowledge and it would amount to accepting the claim that knowledge entails (at least) approximate truth. Since the defenders of KNPR have already adopted what looks like a view that is a radical departure from what is epistemological orthodoxy, they could simply embrace this consequence. The second way to potentially dodge the Robin cases without challenging the adequacy of construal of the relationship between the theories involved and without challenging the assumptions about the concepts of truth and approximate truth made in those cases involves weakening KNPR-N' to require only that the conclusions of practical deliberations be known and the epistemic principle known as safety.[37] The safety condition is often stated as follows: if S believes that p, then p would not easily have been false.[38] Safety is widely supposed to have a strong degree of

[35] See Oddie, "Truthlikeness."

[36] See Oddie, "Truthlikeness" and Kuipers, *What is Closer-to-the-truth?*.

[37] This strategy was suggested by a helpful referee.

[38] See for example Ernest Sosa, "Skepticism and Contextualism," *Philosophical Issues* 10 (2000): 1-18.

intuitive support and so its application here is at least *prima facie* promising. Given this approach, it would be rational to employ false but approximately true premises in practical deliberation provided that the conclusion reached is one that could not easily have been false. The encroacher could then argue on the basis of the safety condition on knowledge that Robin's belief that the arrow would fly further than 30 yards could not easily have been false, because it was derived from theoretical claims that are close to the truth. So, despite the fact that Robin's calculations involve approximately true premises, the conclusion would be known nonetheless.

Let us address these two responses in turn. First, what can we say about the suggestion that encroachers adopt the view that knowledge entails approximate truth rather than truth? As we have already seen this amounts to the denial of factivity and thus entails that at least some falsehoods can be known. There are few more firmly entrenched orthodoxies in epistemology than factivity and so this is a radical suggestion to say the least. It is especially problematic in that it not only has the implication that falsehoods can be known, but also it has the implications that justification cannot exclusively be a matter of support for truth and that belief cannot be commitment to the truth of a proposition. Justification cannot be support for truth alone if factivity is ceded because otherwise some known propositions would turn out to be unjustified. Such propositions would be known approximate truths for which there is no justification in the sense of support for their (strict) truth. So the encroacher who responds in this way would have to replace the standard justification condition on knowledge as well with something more akin to the requirement only that S be justified in the belief that p is approximately true. Similarly, if factivity is ceded, then the commitment involved in knowledge cannot be belief in the (strict) truth of a proposition. This is problematic in both cases because it is widely accepted that truth is the norm both of belief and of justified belief. In ceding factivity, encroachers would have to adopt a view of belief and justification that involves only the commitment to approximate truth. Otherwise some propositions would be known but not believed to be true. As a result, the denial of factivity is dangerously close to collapsing knowledge into mere belief and thus obliterating any possibility of usefully articulating *epistemic* conditions on the rationality of practical reasoning. This seems to be an excessively radical step to take in order to preserve KNPR in light of the Robin counter-examples. In addition, it is not clear that this response works in the case of ROBIN 2. This is simply because in that case Robin does not believe that Newtonian mechanics is approximately true and so cannot

presumably know the propositions that make up that theory, whatever one says about factivity.

Let us then consider the second suggestion involving the safety condition and the weak requirement that only the conclusions of practical reasoning be known. We might state this alternative as follows. Where p is a reason and q is a conclusion drawn on the basis of p and the choice is p-dependent and q-dependent,

> (SCNPR-N) It is epistemically rational for S to employ p and q (appropriately) in S's practical reasoning → (it is at least the case that JBsp is approximately true), (p is at least approximately true) and K*sq.

Here we are to understand that K* sp specifically requires meeting the safety condition. As we have seen this response looks promising because in ROBIN 1 and ROBIN 2 Robin could be taken to know the conclusion of his practical deliberations based on the false but approximately true propositions that constitute Newtonian mechanics because that conclusion could not easily have been false given the approximate truth of that theory. Given this approach, he knows that the arrow will fly further than 30 yards because that belief is safe and that belief is safe because it was derived from propositions that are approximately true. The problem with this view however is straightforwardly clear. One can be epistemically rational in one's practical reasoning even if the conclusion one draws on the basis of approximately true premises is not known. This can be because some such conclusions are approximately true but not safe, and therefore not known according to such views. Consider the following modification of ROBIN 4:

> ROBIN 5: suppose that Robin is an independently wealthy carpenter in 2009 and who is totally ignorant about archery. Suppose then that Robin has been offered a fairly standard sort of performance wager. The terms of the wager are as follows. Robin will be given a bow and arrow, although Robin has never previously used this bow or one of its type. Nevertheless, he is aware of the strength of the bow and is assured that it is perfectly functional, that the arrow is perfectly normal and that anyone can use it effectively without much training. He is allowed to train as much as he likes, however. Robin is asked to put up $50, and provided he can shoot an arrow beyond a marker set at 100 yards he will win $100. If he fails to do so, then he loses the $50 he put up. So, Robin must use his practical reason to determine whether he should accept the wager or not. Initially, given his ignorance of archery, he has no idea whether the bow is capable of shooting an arrow beyond the marker. So Robin takes 10 practice shots and uses his handy (and previously reliable) tape measure to determine to the closest tenth of a foot that arrows landed 100.1 yards, 100.0 yards, 100.1 yards, 100.3 yards, 100.2 yards, 100.2 yards, 100.0 yards, 100.1 yards, 100.2 yards,

> and 100.2 yards away. So, on the basis of what he takes to be approximate measurements he concludes that an arrow fired from the bow will travel at least 100 yards. So, Robin completes his practical deliberation, accepts the wager and proceeds to win, thus doubling his stake.

In this case Robin's practical reasoning is epistemically rational, it is successful and it is based on approximately true propositions that he is justified in believing to be approximately true. But his conclusion could easily have been false because of the small degree of difference between the approximate measured values and the real distance of the marker. So, given safety, Robin does not know that the arrow will travel at least 100 yards. The only way that the appeal to the safety condition can then save KNPR and pragmatic encroachment in general here is by rejecting factivity. But we have already seen the problematic consequences of pursuing that line of reasoning. Consequently, adopting this modification of KNPR is not sufficient to save the pragmatic encroacher's views. Moreover, even if this weakened view could be salvaged it would be a serious concession on the part of the pragmatic encroachers to adopt it in any case, because it is no longer a pure analysis of knowledge in much the same way that Neta's and Littlejohn's proposals are not versions of strong pragmatic encroachment.

7. Conclusion

So it seems to be the case that we actually reason rationally and perform remarkably well on the basis of approximations and JBATNPR best reflects these facts as an account of the epistemic dimensions of practical reasoning. There then is a perfectly well understood sense in which Robin's behaviors in the various Robin cases are rational, but not in the way assumed by the pragmatic encroachers. It seems reasonable to suppose that in the cases described above Robin uses approximations in reasoning about whether to accept the wager in a perfectly rational way even though they are not strictly true. In general we use these sorts of approximations because they are appropriate in specific contexts and in the Robin cases doing so allows him to secure an efficient and successful solution to his problem in a rational manner. He is rational in those cases because he is either justified in believing the relevant claims are true or he is justified in believing that they are approximately true, and those claims really are at least approximately true for those situations. Of course, this would not necessarily be the case for other situations, but this sort of behavior looks to be the norm rather than the exception.

RETHINKING THE *A PRIORI/A POSTERIORI* DISTINCTION

Jennifer Wilson MULNIX

ABSTRACT: This paper offers an account of the *a priori/a posteriori* distinction utilizing the insights of reliabilism, focusing on the *inputs* to reliable belief-forming processes. I propose that a belief possesses *a priori* justification if it is the result of a reliable belief-producing process whose input is 'non-sensory' and the reliability of this process does not 'causally depend' on the reliability of a prior process taking in 'sensory' input. One of the interesting consequences of this account is in the treatment of introspective knowledge of one's belief-states; it was classically considered *a posteriori*, but comes out *a priori* on this model.

KEYWORDS: epistemology, reliabilism, *a priori*, *a posteriori*, naturalism, externalism

1. Introduction

> There is only one way of knowing, the empirical way that is the basis of science (whatever way that may be). So I reject *a priori* knowledge.[1]

This quote by Michael Devitt illustrates a common charge against naturalists (or by naturalists) that *a priori* knowledge is not possible given a naturalistic scientific framework.[2] This assumption is incorrect. One can consistently hold a reliabilist

[1] Michael Devitt, *Coming to Our Senses* (Cambridge: Cambridge University Press, 1996), 2.

[2] By a 'naturalist' I mean an epistemologist who 'reduces' epistemic concepts to nomological concepts. The nature of the reduction is not the same for all naturalized epistemologists. For some, epistemological questions are replaced with psychological questions and the autonomy of epistemology is threatened (Quine); for others, the nature of the reduction is weaker. Under Goldman's view, for example, the concept of normative justification supervenes upon nomic properties. See W.V.O. Quine, "Epistemology Naturalized," in his *Ontological Relativity and Other Essays* (New York: Columbia University Press, 1969), 69-90; Alvin I. Goldman, "What is Justified Belief?," in *Justification and Knowledge*, ed. George S. Pappas (Dordrecht: D. Reidel, 1979), 1-24. For more on the debate over *a priori* knowledge with respect to naturalism, see also Albert Casullo, *A Priori Justification* (Oxford: Oxford University Press, 2003); Michael Devitt, "Naturalism and the *A Priori*," *Philosophical Studies* 92 (1998): 45-65; Georges Rey, "A Naturalistic *A Priori*," *Philosophical Studies* 92 (1998): 25-43.

naturalistic epistemology and accommodate the existence of *a priori* knowledge, that is, knowledge which is in some sense 'independent of experience.'[3]

The classical treatment of the distinction in terms of special types of internal processes is simply unsatisfactory. The mysterious workings of a 'faculty' of reason or intuition are no longer plausible as philosophical explanations for the source of our *a priori* knowledge.[4] This paper's primary focus is to explicate a new substantial account of the *a priori/a posteriori* distinction within a naturalistic framework, an account that will incorporate the insights of a reliabilist epistemology and will disambiguate the murky concept of 'independence from experience' offered by the classical conception. Because the *a priori/a posteriori* division is an epistemic distinction concerned with identifying two fundamentally different sources of knowledge, the most natural interpretation of the distinction for a reliabilist should focus on the *inputs* to belief-forming processes.[5]

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[3] Reliabilist naturalism, as I shall use this term, means that justification is to be understood in terms of reliable belief-producing mechanisms. Epistemic justification is a 'natural' concept—"a function of the psychological [evolved] processes that produce and preserve belief" (Alvin I. Goldman, "*A Priori* Warrant and Naturalistic Epistemology," *Philosophical Perspectives* 13 (1999): 3). The aim of a reliabilist naturalistic epistemology is to evaluate our cognitive capacities by understanding how they non-accidentally bring about beliefs in accordance with the facts. I wish to defend a moderate naturalism, where epistemology needs 'help' from science as evidence for beliefs, but non-empirical warrant is available. For instance, cognitive psychology can 'help' reliabilism in discovering appropriate types of belief-forming processes.

[4] I do not mean to suggest that I am denying 'intuition,' but, perhaps, just denying the suggestion that it is a special 'faculty' (to be explained shortly). I want to argue that 'intuition' might simply be reducible to certain belief-forming processes in the brain. See Goldman, "*A Priori* Warrant and Naturalistic Epistemology"; Ernest Sosa, "Minimal Intuition," in *Rethinking Intuition: The Psychology of Intuition and Its Role in Philosophical Inquiry*, eds. Michael R. DePaul and William Ramsey (Lanham: Rowman & Littlefield, 1998), 257-270. Well, I am also denying the claim that *a priori* justification is *parasitic upon* this faculty of rational intuition.

[5] I do recognize that the approach in terms of characterizing *a priori* processes is a plausible way to understand the distinction, but I think that characterizing the distinction through inputs may prove to be more plausible. The motivation for this conclusion is the following: Belief-producing processes are most likely either functional states or mechanisms. As functional states, the processes themselves are defined in terms of their inputs and outputs. One cannot isolate a belief-producing process, then, without identifying the relevant inputs and outputs. Thus, the question of whether the distinction should be drawn according to *a priori* processes or *a priori* inputs would turn out to be the same question. As mechanisms, however, inputs may be separable from processes, depending on how we decide to type our processes. In this case, it could turn out that the same process type sometimes takes in sensory inputs and at other times takes in non-sensory inputs. In such cases, the process would not be a 'special *a priori* process,' but it would nonetheless produce *a priori* justification. This is issue is quite complex and must face the Generality Problem, but consider the following example: I may

I should warn the reader at the outset that some of our knowledge will not be classified in the same way on my conception of the *a priori/a posteriori* distinction as on the classical model. On my view, some knowledge previously thought to be *a posteriori* comes out *a priori*. There is a cost to precision. The virtue of this account will be its ability to draw a clear distinction between the *a priori* and the *a posteriori* according to 'sensory' versus 'non-sensory' inputs.[6] What is meant by 'sensory' and 'non-sensory' will be analyzed shortly. But, it is not a fault of *analysis* that there are disputes regarding the sensory/non-sensory status of particular mental states. It may require the aid of neuroscience to determine whether the input of a given belief-producing process is sensory or non-sensory. It is a *virtue* of this account that it allows us to draw the *a priori/a posteriori* distinction in terms of inputs to processes while recognizing that there is a legitimate debate over whether a given experience is sensory or non-sensory.

The '*a priori*' may refer to knowledge, justification, propositions, or the way concepts are acquired. In this paper I will not be concerned with *a priori* concepts or concept acquisition, and so I will omit a discussion of innate concepts or 'nativism.'[7] The *a priori/a posteriori* division is *at base* an epistemological distinction, and is only derivatively connected with the modal status of propositions. My purpose is not to find a certain class of truths that may be particular to the domain of *a priori* knowledge. My focus will be on how *a priori* justification differs from *a posteriori* justification, but what I say regarding

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have a belief-producing process of introspection, where in one case the input to my introspective process is a particular pain – a sensory mental-state – and at another time it is a particular seeming-to-remember – a non-sensory mental-state. Presuming, of course, that these are both tokens of the introspection process type, then we have a case of a process that yields both *a priori* and *a posteriori* justification – it is not a 'special *a priori* process' but produces *a priori* justification. My account advocates a concept of minimal apriority, making no further distinction for the 'pure' *a priori*, as Kant did. Rather, sometimes the same belief which possesses an *a priori* justification may also possess an *a posteriori* justification. The fact that the belief may also possess an *a posteriori* justification in no way nullifies its separate *a priori* justification. That said, searching for special *a priori* processes appears to rule this possibility out of hand.

[6] For reasons of convenience, I refer to the inputs producing *a priori* and *a posteriori* justification respectively as, 'non-sensory' and 'sensory,' rather than using the terms 'non-experiential' and 'experiential.'

[7] Nativism is the view that we are all possessed not only with innate structures and capacities, but also with innate information, ideas, concepts, beliefs, and even knowledge. Nativism is an explanation of the origin of our beliefs, and not a theory of justification.

justification will apply *mutatis mutandis* to other epistemic concepts. *A priori* knowledge is to be defined as belief that is true and carries *a priori* justification.[8]

What is the classical analysis of *a priori* knowledge? Traditionally, the view that we have *a priori* knowledge was associated with rationalism. The rationalist asserts that, "*a priori* justification occurs when the mind directly or intuitively discerns or grasps or apprehends a necessary fact about the nature or structure of reality."[9] According to rationalism, intuition is needed in order to explain how we possess certain concepts (e.g. the concept of infinite perfection) and how we are able to 'grasp' the necessity of certain truths (e.g. truths about the properties of a triangle) – knowledge of these truths requires more than experience can provide. Pure intuition was considered a clear case of a process, available independently of experience, able to produce *a priori* knowledge. The moderate empiricists, on the other hand, though conceding the existence of *a priori* knowledge of analytic truths, in effect undermined its significance by reducing the object of our *a priori* knowledge to a mere relation among ideas.[10]

The *classical* analysis took the necessity of truths known to be the mark of the *a priori*, making a proposition knowable *a priori* only if it is necessarily (and analytically) true.[11] Kant (disputably) widened the scope of the *a priori* to include synthetic truths, but these truths were still necessary. It wasn't until the influence of Kripke that necessity became separable from the conception of the *a priori*. Many disagree, however, with Kant's analysis of synthetic *a priori* truths, citing in the final analysis an underlying analyticity.[12] Kripke's analysis of the contingent *a priori* is also not uncontroversial, as it is dependent upon certain contentious considerations regarding the ways in which one can 'fix the reference' of

[8] Although possibly also needing to satisfy further knowledge conditions. Refer to the Gettier literature for discussion of this issue.

[9] Laurence BonJour, "A Rationalist Manifesto," *Canadian Journal of Philosophy Supplement* 18 (1992): 56.

[10] As opposed to the radical empiricists, such as Quine, who deny the possibility of *a priori* knowledge altogether. Moderate empiricism would include philosophers such as Hume, Kant, and Ayer.

[11] Goldman explains that the classical conception of the *a priori* has included the following characteristics: "(1) Necessity [of what is known *a priori*]; (2) Non-sense-experiential source or basis [of one's justification]; (3) A subject-matter of abstract eternal objects [for propositions knowable *a priori*]; (4) Incorrigibility (rational unrevisability) [of one's justification]; (5) Certainty [possessed by one who has *a priori* justification]; (6) Infallibility [of one's justification]" (Goldman, "*A Priori* Warrant and Naturalistic Epistemology," 4-5). Of these concepts, my account needs only to retain (1) – a non-sense-experiential source or basis. Goldman, on the other hand, retains both (1) and (2) and rejects (3)-(6).

[12] See BonJour, "A Rationalist Manifesto," Section 3 for one form of the charge.

expressions referring to kinds. Any account which attempts to expand the class of truths known *a priori* beyond the analytically necessary must answer the question: "How can a proposition be known (specifically, justified) *a priori* if it is true in virtue of considerations other than the meaning of its constituent terms?"[13]

2. A Reliabilist Account of the *A Priori/A Posteriori* Distinction

My account can provide such an answer. To begin, a belief carries *a priori* justification if the truth of the belief could be ascertained independently of experience.[14] This is also how the traditional account begins. But proponents of the classical view were quick to see that *a priori* knowledge is not to be understood as belief justified independent of *any* sort of experience whatsoever, for then we could not have any *a priori* knowledge! Even the knowledge of a basic mathematical truth involves some type of experience (e.g. the contemplation of numbers), but neither the classical conception nor my own would characterize the justification as *a posteriori*. Thus, all accounts of the *a priori* must search for the

[13] Paul K. Moser, *A Priori Knowledge* (Oxford: Oxford University Press, 1987), 6. I am making the assumption here that if a proposition is true in virtue of the meaning of its terms, then it is analytic, and that all analytic truths are necessary truths. David Kaplan, on the other hand, separates the notion of analyticity from necessity, arguing that one can have *a priori* knowledge of an analytic proposition which is not necessary. See David Kaplan, "Demonstratives: An Essay on the Semantics, Logic, Metaphysics and Epistemology of Demonstratives and Other Indexicals," in *Themes from Kaplan*, eds. Joseph Almog, John Perry, and Howard Wettstein (New York: Oxford University Press, 1989), 481-564. For the purposes of this paper, however, I am not at all concerned with providing an analysis of what makes a proposition analytic versus synthetic or whether analyticity entails necessity.

[14] Of course, experience is minimally needed for concept acquisition, but as indicated earlier, I am only interested in the *epistemic* distinction between the *a priori* and *a posteriori*. There is also the further point that sometimes sensory-experience is required in order to have justification (e.g. an arithmetic or geometric proof), but that does not make the sensory-experience part of the justification. Causally necessary conditions are not always relevant to justification. Certain experience may be a precondition for coming to know a truth, but that experience may not play a justificatory role in ascertaining the truth of the proposition. Arguably, perception of visual shapes or objects may be required to 'trigger' or 'occasion' our apprehension of mathematical truths, though not playing any fundamental justificatory role (such as, for example, counting pebbles, which cues our apprehension of the truth of "3 + 2 = 5"; or, more controversially, our perception of a diagram, perhaps, serves only as "an heuristic to prompt certain trains of inference," which inferences then justify our belief in the truth of the Pythagorean Theorem). For more discussion of this issue, see for instance, Jaegwon Kim, "The Role of Perception in '*A Priori*' Knowledge: Some Remarks," *Philosophical Studies* 40 (1981): 339-354; Dennis Lomas, "What Perception is Doing, and What it is Not Doing, in Mathematical Reasoning," *British Journal for the Philosophy of Science* 53 (2002): 205-223.

subclass of experience that is relevant to *a posteriori* justification. However, instead of delineating a principled distinction, the tradition integrated diverse sorts of experience, such as sensory-experience and introspection, provided they did not involve 'special' *a priori* rational processes. But this is to miss the point; focusing on types of processes rather than forms of experience fails to define '*a posteriori* experience' and consequently, falls short of giving any illuminating explanation of 'independence from experience.'

On my view, however, the separation can be readily understood in terms of inputs. A non-inferential justification is *a posteriori* if the inputs to the reliable belief-forming mechanism are 'sensory.'[15] Non-inferential *a priori* justification, then, will involve inputs which are 'non-sensory.' What is to count as 'sensory'? There may be no uncontroversial way to distinguish the sensory from the non-sensory given the complicated nature of mental states. One common understanding of the sensory is drawn according to the five senses (the olfactory, gustatory, auditory, visual, and tactile) and the kinesthetic sense. One reasonable analysis, then, places states involving the five senses and the kinesthetic sense into the domain of the sensory and the states which remain into the non-sensory category.

However, it is not clear that pains and emotions, two types of states which we intuitively consider to be sensory, directly involve one of our five senses or the kinesthetic sense. One may then modify the distinction by counting as sensory those states which are non-intentional and non-sensory as those states that are intentional. Understanding the division in this way, however, may again fail to accord with our intuitions if one understands pains and emotions as intentional (or if one understands sensations as intentional). An alternate way to demarcate the categories is to separate those states that are physical from those that are non-physical. This differentiation would account for our pains and emotions but further problems may arise if one takes all mental states to be essentially physical (brain-states). It seems that whatever way the distinction is drawn, we would like to have it end up that our pains, emotions, and perceptions are sensory but that our belief-states and some 'awarenesses' are non-sensory.[16]

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[15] Since *a priori* justification is defined recursively, inferential *a priori* justification will be given a different analysis, as will become clear later.

[16] There may be many different kinds of awarenesses, some of them sensory (such as an awareness of my pain) and some non-sensory. The awarenesses I am concerned with in regard to the non-sensory have to do with grasp of meaning, such as an awareness of a belief-state, numbers, universals, or propositions. What is more, these awarenesses can serve as inputs to belief-producing processes in a way that is consistent with naturalism, for it is the awareness that serves as the input to the belief-producing process, not the numbers or universals themselves. For example, the awareness of a proposition, such as the proposition, 'big snakes

How, then, can we account for our basic intuitions while maintaining a principled distinction? Let's return to the initial characterization involving our sensory organs. Why not consider pain and emotions as directly involving the five senses (and the kinesthetic sense)? One may reasonably argue that pains can be understood on a continuum with basic touch experiences, where pains are understood as certain sorts of extreme cases of tactile sensations.[17] What about our emotions? Emotions are particularly problematic. Some emotions seem fundamentally intentional while others do not; some seem essentially physical while others do not. For example, it is not clear that being in-love, while intentional, is also physical; but feeling angst may be non-intentional and physical. Classifying all emotions as sensory, then, fails to account for their complexity. However, an emotion which has physical sensations as *constituents*, whether intentional or not, is likely to involve the five senses. Consider anger, for example. The fact that we may use certain sensations (e.g. clenching of fists and teeth, tightening of muscles, shortness of breath, flaring of nostrils, etc.) to identify that we are feeling angry is not relevant to its status as a sensory or non-sensory mental state – *but*, if bodily sensations are partly *constitutive* of the state of anger, then the state should obviously be classified as sensory. Other emotions appear to fit with this model.[18] Belief-states and awareness of numbers, propositions, or belief-states, on the other hand, do *not* seem to involve the stimulation of our five senses, even when broadly understood as involving a continuum. This, then, is the proposal for understanding the sensory and the non-sensory. I have tried to present various ways one can understand this division, but I certainly invite others to provide an alternative distinction and presuppose it in what follows.

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are dangerous,' might be an input (not the proposition itself) to one of my belief-producing processes, which outputs a belief in the truth of the proposition.

[17] For instance, consider George Berkeley's discussion in the *Three Dialogues between Hylas and Philonus*. At one point in the First Dialogue, Philonus suggests an identification of extreme heat with extreme pain thought of as a tactile sensation: "Seeing therefore they are both immediately perceived at the same time, and the fire affects you only with one simple, or uncompounded idea, it follows that this same simple idea is both the intense heat immediately perceived, and the pain; and consequently, that the intense heat immediately perceived, is nothing distinct from a particular sort of pain." This suggestion was given to me by Richard Fumerton in conversation.

[18] Some emotions, then, will be non-sensory if they do not involve the stimulation of the five senses. An example of an emotion of this type might be being in-love. Let us call these emotions 'emotion$_2$.' Although recognizing these two types of emotion, the use of 'emotion' in this paper refers only to those which essentially involve – are constituted by – states involving the direct stimulation of the five senses ('emotion$_1$').

Returning to our analysis of justification, my formulation of how a belief could possess *a priori* justification is remarkably *simple* – 'independence from experience' is to be determined solely according to the sensory/non-sensory nature of the inputs to the reliable belief-producing process, *whatever process it may be*, and not according to special '*a priori* processes'[19] (with one further complication to be explained shortly – one also has to say something about the origin of reliability). One can have a fully adequate definition of the *a priori* in terms of inputs to reliable processes without including as part of the *definition* identity conditions for processes across counterfactual situations (though an analysis of *justification* in terms of reliability might include such considerations to help discover whether a given process is justified).

I hold a foundational account of knowledge where non-inferential knowledge, which is to serve as the foundation, is to be understood as justified true belief produced from an unconditionally reliable belief-independent process *or* from an unconditionally reliable *but belief-dependent* process.[20] As I will explain shortly, introspective belief is a paradigm case of a belief formed from an unconditionally reliable but belief-dependent process. This formulation of non-inferential knowledge differs in some significant respects from the standard reliabilist formulation as given by Goldman,[21] but is one, I believe, that best captures the reliabilist's commitments, and is something which has been to my knowledge previously overlooked. That is to say, a process does not need to be belief-*in*dependent in order to be unconditionally reliable. A process is unconditionally reliable if a sufficient proportion of its output beliefs are true. Inferential knowledge, on the other hand, is justified true belief produced from a conditionally reliable belief-dependent process whose input beliefs are themselves justified. A process is conditionally reliable if, given that the input beliefs are true, a sufficient proportion of its output beliefs are true.

Applying this analysis of non-inferential justification to our account of the *a priori*, we see that a belief is non-inferentially *a priori* justified if the inputs to the unconditionally reliable belief-producing process are non-sensory. Any process,

[19] To be clear, by this I do *not* mean that reliability is to be understood instead in terms of process tokens rather than types. It may also be possible for the same process to take in both sensory and non-sensory inputs allowing the belief produced to carry both types of justification, although my view is not committed to this claim. This claim would seem to require belief-producing processes to be defined as mechanisms. If belief-producing processes are best understood as functional states, then this will not be the case.

[20] There are, of course, Gettier considerations one needs to take into account. See the Gettier literature for difficulties surrounding Gettier cases.

[21] See Goldman, "What is Justified Belief?"

then, which is unconditionally reliable and which takes as its immediate input something non-sensory would yield a non-inferentially *a priori* justified belief.

3. A Note on Reliability and 'Causal Responsibility'

Let us pause here for a moment. This analysis appears to get things correct with regard to our mathematical knowledge. Consider the belief that 2 + 2 = 4. This belief carries non-inferential *a priori* justification, the inputs to the process being the contemplation of numbers or propositions. But what about the following case? Consider the possibility that we are so constituted such that we have a reliable belief-producing mechanism where usually, whenever our sole input is a seeming-to-remember that some event *x* occurred, our output is a belief that *x* occurred. This type of input seems to be a case of a non-sensory non-doxastic intentional state, which makes the resultant justification *a priori*. An example might be the true belief that I locked my door yesterday where the input is a seeming-to-remember that I locked my door yesterday. Even though these seemings-to-remember involve sensory experiences, the seeming-to-remember itself is the sort of state which seems non-sensory; yet do we really want it to be the case that my belief that I locked the door yesterday is non-inferentially *a priori* justified?

There may be several ways to avoid this conclusion. One is to deny that seemings-to-remember are non-sensory states, but to argue this would require detailed analysis.[22] An alternative explanation might suggest that beliefs about the past actually involve other beliefs as inputs (e.g. I was holding my key, I was standing in front of my door and not someone else's, I turned the lock clockwise, etc.), thus rendering the justification inferential and *a posteriori*, and for that reason, avoiding our potential difficulty (because beliefs which are justified through a combination of sensory and non-sensory inputs are *a posteriori* justified).

Perhaps the most plausible way to address this worry is to focus on the alleged reliability of the process taking in seemings-to-remember and outputting beliefs about the past. Why is it the case that these beliefs about the past are reliable? Recall that in the original description of the scenario, this belief-producing process does not take in beliefs as input, but rather, solely non-sensory non-doxastic states. It is purported to be unconditionally reliable – the relevant definition of reliability makes no reference to the truth-values of the inputs (if

[22] The idea here is that seemings-to-remember, in the sense that they might involve image or video replay in one's mind, might be more like episodic memory than stored beliefs. Interestingly, for Hume, beliefs, impressions, and imaginings were only different in degree, and not in kind. On his view, however, it would be quite difficult to separate out the non-sensory from the sensory for this reason.

there are any). But in what sense would we consider this to be the case without the existence of some prior reliable process involving sensory experiences? To be more precise, if the prior process which took in a sense-experience and outputted the belief that I locked my door was not reliable, then the latter process involving seemings-to-remember would not be reliable – seemings-to-remember could just pop into my head randomly. Thus, the *reliability* of the process involving sense-experience is *causally responsible* for the reliability of the process taking in seemings-to-remember, in the sense that it causally contributes to the latter process's reliability (rather than just being *causally necessary* for the process to begin).[23]

We may then wish to add to our analysis a conceptual distinction between being 'causally necessary' and 'causally responsible.' On the one hand, the existence of process *P* may be 'causally necessary' for the initiation of process *Q*. An example of this might be evolution. It might be the case that, without our species having undergone certain evolutionary processes, we could not engage in other sorts of belief-forming processes, such as perception. So, we might say that a former process is 'causally necessary' for a latter. But, we can also speak of a process being 'causally responsible' for another in the sense that the *reliability* of process *P* causally contributes to the *reliability* of process *Q*.[24] In our case above involving memory, had there not been a prior sensory process *P* that was reliable, then my process *Q* involving beliefs about the past from seemings-to-remember might be entirely unreliable. We would say, then, that the reliability of my prior sensory process is 'causally responsible' for the reliability of my later memorial process.

As an additional illustration, take the case of perception and the contemplation of numbers. It might be the case that, without having had certain perceptual experiences, we would not be able to form beliefs about numbers resulting through contemplation of them. But, the reliability of the perceptual process does *not* causally contribute to the reliability of the process which results in beliefs about numbers from our contemplation of them, since our perceptual mechanisms could be wholly unreliable while our process outputting mathematical beliefs is reliable. Thus, in our stipulated example, though process *P* might be 'causally necessary' for process *Q*, it is not 'causally responsible' for

[23] The prior *a posteriori* process generates the reliability of the latter process, so the justification for the deduced belief is derivative or 'inherited.'

[24] This will then also give us a notion of 'causal dependence': The reliability of a process will 'causally depend' on a prior process when the reliability of the former process is 'causally responsible' for the reliability of this latter process.

process *Q* because the reliability of process *Q* is not derived or 'inherited' from process *P*.

Modifying the analysis, then, a belief is non-inferentially *a priori* justified if the inputs are non-sensory and the process's unconditional reliability does not 'causally depend' upon a prior process taking in sensory inputs; a belief is non-inferentially *a posteriori* justified if the inputs to the process are sensory *or* the inputs are non-sensory but the process's unconditional reliability does 'causally depend' upon a process taking in sensory inputs, in the sense that the reliability of the prior process is 'causally responsible' for the latter process's reliability.

Again, take our example involving perception and the contemplation of numbers above. Suppose that I believe that 2 + 2 = 4, and I formed the belief initially by observing groups of apples on a desk. Later, I have the belief that 2 + 2 = 4 through the contemplation of numbers. This belief is *a priori* justified because the inputs are non-sensory *and* because the process involving the contemplation of numbers does not rely upon the *veridicality* of sense-experiences in order for the belief to be justified; the process involving contemplation is reliable even if I am dreaming about or hallucinating apples – if my perceptual mechanisms are wholly unreliable. So, the *reliability* of the process involving sense-experience is *not* causally responsible for the *reliability* of the process taking in contemplation of numbers, and so the *a priori* justification is preserved.

Some, such as Kant and Kitcher, are disposed toward making a division between partially *a priori* justified beliefs and purely *a priori* justified beliefs, where the former beliefs also involve an *a posteriori* justification. In my analysis, a belief which is justified both through sensory input and through non-sensory input is *a posteriori* justified. An example of this might be the following: I may have certain geometric beliefs which are justified through *a priori* chains of reasoning, but it turns out that the visual shapes on the paper are also playing an important justificatory role by serving as inputs to my belief-producing process. In this case, rather than saying my belief in a geometric theorem is partially *a priori* justified and partially *a posteriori* justified, it would simply be *a posteriori* justified by virtue of involving any type of sensory component in the justification. Moreover, even if a belief is reliably formed from a sensory input (and so carries an *a posteriori* justification), if the same belief could also be reliably formed without that, or any, sensory input, the belief *could* also possess an *a priori* justification.

Understanding, then, that non-inferential knowledge can result from an *unconditionally* reliable belief-*dependent* process, we can easily note two important distinctions: (1) Non-inferential belief-producing mechanisms can take

beliefs as their inputs since the justificatory status of the input beliefs is independent of the process's tendency to produce true beliefs (because the relevant definition of reliability makes no reference to the truth-values of the inputs, if there are any) – an introspective belief about a belief is a paradigm; and (2) For a belief to be inferentially *a priori* justified, it is not sufficient that the input to the belief-producing mechanism be another belief (for then all inferential knowledge would be *a priori* justified!). What is also required is that the justification of the output belief ultimately depends upon beliefs which are themselves non-inferentially *a priori* justified. Like a theory of justification, then, *a priori* justification is defined recursively.

To make clear this first distinction, consider basic mathematical knowledge, such as the belief that 2 + 2 = 4. This belief carries non-inferential *a priori* justification, the inputs to the process being contemplation of numbers or propositions. But, a more striking consequence of this account is that it will turn out that much of our 'introspective' knowledge carries non-inferential *a priori* justification. Consider the belief that I have the belief that I am in pain. The unconditional reliability of the process outputting the second-order belief about my belief is not established through the justificatory status of the first-order belief. Complex mathematical knowledge, on the other hand, holds *inferential a priori* justification because the input beliefs to the belief-producing mechanism yielding the output beliefs *must be justified* (e.g. a belief about theorems of Euclidean geometry).

Regarding the second distinction, the belief that it rained this morning is an example of a belief with inferential *a posteriori* justification because its justification is ultimately owed to some beliefs with *a posteriori* justification (e.g. I see water on the streets, I heard what sounded like raindrops, etc.). A belief about the Pythagorean Theorem, on the other hand, possesses inferential *a priori* justification because its justification ultimately depends upon beliefs which are *all* non-inferentially *a priori* justified (e.g. axioms of geometry).

This raises the question as to how to characterize the relevant inputs. Specifically, how far back in the causal chain should we go in characterizing a belief-producing process (and so, to identify the relevant input)? It is important in characterizing inputs not to go back indefinitely far. Conee and Feldman explain that views differ over how to understand the relevant input. One might argue that the input "begins at the surface of the skin, or farther in at some point where conscious experience begins, or farther out in an external cause of the experience."[25] Alston's position places the relevant inputs in the middle of the

[25] Earl Conee and Richard Feldman, "The Generality Problem for Reliabilism," *Philosophical*

spectrum offered by Conee and Feldman; in particular, in cases of visual belief-formation, the process begins with the perceptual experience.[26] Kitcher, as well, claims that the beginnings of the causal sequences constituting the processes should be restricted to those segments which consist solely of states and events internal to the believer. Goldman has a similar view, maintaining that only the "proximate causes internal to the believer" are constitutive of the belief-producing process.[27] In agreement with these philosophers, my own view is that the inputs of the causal sequence constitutive of the process are restricted to those states internal to the subject.[28] Of course, a correct specification of the 'relevant type' of process involved is also required.[29]

Studies 89 (1998): 27, n. 1.

[26] Alston explains the reasoning for his position: "If the epistemic status of a belief is a function of the reliability of the process that generates the belief, it is the reliability of the psychological process that is crucial. Looking at perceptual belief formation, no matter how exemplary the path of the light rays from the surface of the perceived object to the retina, and no matter how finely tuned the neural transformations involved in the pathway from the eye to the brain, if the belief is not formed on the basis of the conscious presentations (and/or its neural correlate) in a truth-conducive way, the belief will lack the epistemic desideratum that is stressed by reliabilism" (William Alston, "How to Think About Reliability," *Philosophical Topics* 23 (1995): 12).

[27] See Goldman, "What is Justified Belief?" and *Epistemology and Cognition* (Cambridge: Harvard University Press, 1986).

[28] However, Fumerton, in conversation with me, has expressed a worry that restricting the relevant inputs to those that are internal to the subject fails to account for the truth-conducive character of certain processes, such as those involving seemings-to-remember.

[29] Specification of the process is important in determining whether the belief carries *a priori* justification. I am well aware of the Generality Problem for reliabilism in all of its degrees, but there is not room here to address this issue. One possible outline for determining the relevant type is given by Alston: "The relevant type for any process token is the natural psychological kind corresponding to the function that is actually operative in the formation of the belief" (William Alston, *Beyond 'Justification': Dimensions of Epistemic Evaluation* (Ithaca: Cornell University Press, 2005), 126). Alston acknowledges that a process token can be a member of indefinitely many types; nonetheless, some of the types are "ontologically rooted, fundamental, and important in ways many others are not." Alston thinks there is something like a 'natural kind type' for each process token, which is its *function*, and where the 'relevant type' would include all and only those process tokens with the same *causal registering* features. Refer to Conee and Feldman, "The Generality Problem for Reliabilism" for a detailed discussion of the Generality Problem.

4. Introspection and the *A Priori*

How does my view compare with the classical analysis? One fundamental difference rests in the treatment of 'introspection.' Does introspection count as sensory or non-sensory? Are introspective beliefs *a posteriori* justified? While these questions have posed significant difficulties for the classical appraisal, they can easily be answered under my account by reducing 'introspection' to certain belief-forming mechanisms. Thus, whether or not introspection is to be counted as *a posteriori* will be determined by examining the particular belief-forming mechanism's inputs.[30]

Sense-experiences, emotions, and pains would be inputs yielding *a posteriori* justified introspective beliefs. For example, a sense-experience may be an input to a process outputting a belief about the physical environment, *or* to a process outputting a belief that I am having that sense-experience, the latter producing an *a posteriori* justified introspective belief. Belief-states and awareness of numbers would be inputs yielding *a priori* justified introspective beliefs. For example, a belief that the Padres won yesterday may be an input for a process yielding the belief that the Padres will win the pennant, *or* to a different process yielding the belief that I believe that the Padres won yesterday, the latter being an *a priori* justified introspective belief. It is *a priori* justified because the input to the unconditionally reliable process is *not* the sensory experience of watching the Padres win but the *belief-state*, and the *justification* of the second-order belief does not depend on the sensory-experience.

Notice here that under this account, introspection does not involve a (*sui generis)* non-doxastic awareness of the belief-state, which serves as a mediator between the belief-state and the second-order belief.[31] Simply, introspecting a belief-state is *nothing more* than a process taking belief-states as inputs and outputting second-order beliefs about those belief-states. The belief is also non-inferentially justified because although the input into the process is a belief-state, the justification of the outputted second-order belief is independent of the epistemic status of the first-order belief – the belief-producing mechanism is unconditionally reliable. Even if the belief that the Padres won is false, my belief that I have that belief is justified.[32]

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[30] What is particularly nice about this account of introspection is that there is no need to rely simply on analogies to other kinds of experience, as the classical tradition did, to determine whether the justification of various introspective beliefs is *a priori* or *a posteriori*.

[31] This account of introspection is only one among the standard models.

[32] The relation of the *a priori* to introspection receives particularly careful discussion in the debate over the compatibility of semantic externalism with self-knowledge. For more

What might be some other possibilities for contingent *a priori* beliefs under this view? Evolution *may* have predisposed us such that upon mere contemplation of certain propositions, they are immediately believed.[33] Consider the case of contemplating the proposition "Big snakes are dangerous." The input into the process is the contemplation of the proposition and the output is a belief in the truth of the proposition.[34] Other *possible* inputs[35] into this type of process may be the contemplations of "There is life after death," "Most things have causes," or "I exist."

This account does make it, in principle, possible for any belief to have *a priori* justification, provided it can be formed reliably from a belief-producing process taking in non-sensory inputs. Even so, it is important to keep in mind what belief-forming processes are in point of fact going to turn out reliable in the actual world. One could hypothesize a case in which I have a process that takes as input the sensory-experience of a table and outputs the belief that 2 + 2 = 4. If I am in a world where such a process does exist, then it does not seem unintuitive to conclude that the justification is a posteriori. Conversely, a scenario might be envisaged where I now have a process that takes as its input a mathematical belief and outputs a belief that Obama is the current president. By stipulating that the process both exists, and further, is reliable, the justification given to the belief about our president would turn out to be *a priori*.[36]

Comparing this view again with the tradition, although this proposal retains the indispensable constituent of the classical analysis – a non-sensory source for *a priori* knowledge – it peels away most of the traditional characteristics of the *a priori*, such as certainty possessed by one who has *a priori* justification; infallibility of one's justification; indefeasibility of one's justification; and necessity of what is

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discussion, refer to Peter Ludlow and Norah Martin, eds., *Externalism and Self-Knowledge* (Stanford: CSLI, 1998).

[33] An alternative way of characterizing beliefs of this type is to maintain that they result from an innate or genetically endowed reasoning mechanism, where a person has at her use an innate process which enables her to form logically accurate beliefs based on certain inputs. See Goldman "*A Priori* Warrant and Naturalistic Epistemology."

[34] Obviously, here, I am not referring to a reliable process that is the contemplation of *this particular proposition* – "Big snakes are dangerous" – and the belief in it. Nor am I referring to the contemplation of all beliefs, for surely many false beliefs would end up in there.

[35] These are merely various *possibilities* rather than my considered views on the matter.

[36] In other words, when considering odd possible worlds in which there are odd but genuinely reliable belief-producing processes, it should not alarm us that apparently odd beliefs get classified as either *a priori* or *a posteriori* justified respectively. The oddity of these outcomes is not a function of the account, but of the possible worlds we are being asked to consider, and hence, does not necessarily represent a bona-fide counterexample to the position.

known *a priori*. This is allowable because under this reliabilist formation, what makes a belief justified is a simple matter of whether it results from a reliable belief-producing process. Furthermore, one can have *a priori* justification for a false belief – the process might only be slightly reliable.[37] Accordingly, since a reliabilist account of justification does not require the conditions of indubitability or infallibility for a belief to be justified, it follows that these are not necessary conditions for *a priori* justification. What is more, because there is both an inferential and a non-inferential *a priori* justification, self-evidence is also not a necessary feature of the *a priori*. Likewise, it is not clear that a proposition needs to be rationally unrevisable if it carries *a priori* justification. If one allows that *a priori* justifications do not guarantee the truth of the belief, then it is not clear that the possibility of disconfirmation undermines its *a priori* status. As we have seen, these beliefs may be contingent and so it makes sense that the inclusion of additional evidence may change the justificatory status of a belief.

Finally, given my assertion that many contingent beliefs can be known *a priori*, it is critical to disconnect the concept of necessity from *a priori* justification. But, it may be objected that if a person knows that *p a priori*, then "he can know that *p* without any information about the kind of world he inhabits. So, necessarily *p*."[38] If the truth of the proposition depended on a contingent feature of the actual world, the argument goes, then how could one know the proposition without looking? Maybe, the arguer intimates, the actual world is one of the possible worlds where the proposition would have been false. Responding, Kripke explains that this relies upon the presupposition that "there can't be a way of knowing about the actual world without looking that wouldn't be a way of knowing the same thing about every possible world."[39] Moreover, how a proposition can be known does not dictate its modal status. Still, the objector presses, if a belief may be possibly false, how can it be known to be actually true independent of an examination of one's experience? In response, there is nothing outright problematic in having justification for a belief that is false. Secondly, this is a confused way of looking at the justification of a contingently true belief.

[37] Philip Kitcher is on the opposing side, disagreeing with the claim that there can be *a priori* warrant for a false belief. He alleges that *a priori* must be 'ultra-reliable,' guaranteeing the truth of the belief. In point of fact, reliable belief-producing processes are not required to be 'infallible' or 'necessarily reliable' – 100%. A process is reliable if it produces true belief at a "sufficiently great proportion" (perhaps, simply more than 50% of the time). See Philip Kitcher, "*A Priori* Knowledge," *Philosophical Review* 89 (1980): 3-23.

[38] Kitcher, "*A Priori* Knowledge," 17.

[39] Saul Kripke, *Naming and Necessity* (Cambridge: Harvard University Press, 1972), 38.

Justification does not require reliability over possible worlds, but only the reliability of the belief-producing mechanism in the actual world.[40]

As we have seen, the analysis of the concept 'independence from experience' provided by the classical model is vague, at best. The principal advantage of this reliabilist naturalistic account is its ability to make plain this fundamental concept such that it allows for a clean division between the *a priori* and the *a posteriori*. On this view, some knowledge thought to be *a posteriori* comes out *a priori* (e.g. introspective knowledge of one's belief-states). While it is not a new idea to argue that certain contingent propositions can be known *a prio*ri, my view is distinctive in that it allows for this in a different way than Kripke's reference-fixing account. But, it is not the aim of this paper to take a stance on the status of particular – certain possible candidates were suggested in order to illustrate how the contingent *a priori* would be understood on this model. For, the significance of this theory lies not in the range of contingent propositions that can be *a priori* justified (this is only the icing on the cake), but rather in the creation of a new division between the *a priori* and *a posteriori* within a naturalistic framework, its most substantial contribution being that questions as to whether particular beliefs carry *a priori* or *a posteriori* justification is now clearly defined.[41]

[40] One could define reliability according to the actual world, by appeal to counterfactuals that hold true for this world, without needing to appeal to other possible worlds where the counterfactuals for this world are not true (i.e. the antecedent conditions do not obtain). On a possible worlds account of counterfactuals, it will be the closest possible worlds (and not all logically possible worlds) that are referenced for the truth conditions of the counterfactuals regarding reliability of the actual world.

[41] For helpful feedback and conversation, I would like to especially thank Richard Fumerton.

DEBATE

INTERNALIST EVIDENTIALISM AND EPISTEMIC VIRTUE: RE-REPLY TO AXTELL

Trent DOUGHERTY

ABSTRACT: In this brief re-reply to Axtell, I reply to key criticisms of my previous reply and flesh out a bit my notions of the relationship between internalist evidentialism and epistemic virtue and epistemic value.

KEYWORDS: virtue epistemology, evidentialism, reductionism, epistemic value, Meno problem

1. Why I don't think epistemic responsibility deserves its own category: on not multiplying categories without necessity

Axtell asserts that the standard cases of practical irrationality and moral responsibility that I mention seem "very dissimilar in basic respects" to my example of a case of epistemic irresponsibility (which Axtell readily accepts as such). I think that is too strongly stated. Furthermore, I suspect there is a 'philosopher's mistake' in the neighborhood. We philosophers tend to think there is something extra special about the 'quest for truth' when, in fact, it is just one quest among many, and for most of the world it is subordinate to the 'quest for survival' (in the Two Thirds World) or the 'quest for the next hot thing' (in the West). And it could be that there are other apparent dissimilarities due to the fact that I was naturally attempting to illustrate categories with unambiguous paradigmatic instances. Cases more near the borders will seem more similar. Furthermore, I don't know that given a broad array of cases of moral irresponsibility we should expect them all to clearly look alike. In point of fact, the examples I gave are quite diverse and yet Axtell doesn't question them as cases of the same kind. Thus, I don't think Axtell has presented any kind of disconfirmation by the (alleged) disimilarity.

This last point is worth elaborating, for it illustrates a point I've been trying to make all along. The point concerns an appropriate respect for parsimony. Here are three examples from the previous paper, all of which can be instances of moral irresponsibility: forgetting to mail an important check, drinking too much, spending too much on a watch. I said I was trying to give paradigm examples of non-epistemic failings. And Axtell raises no suspicions about my list. Yet what if someone claimed *this* list contained items too diverse to fall under one banner because they were "very dissimilar in basic respects"? The first involves memory,

the second physical health, the third fiscal matters. So perhaps the first involves a new, sui generis form of normativity *memorial irresponsibility*. Perhaps the second involves a new, sui generis form of normativity *bodily irresponsibility*. Perhaps the third involves a new, sui generis form of normativity *fiscal irresponsibility*. That would clearly be absurd. These are all cases of moral irresponsibility in different domains of life. That is *precisely* what I am saying about so-called cases of epistemic irresponsibility. There is no new, sui generis form of normativity *epistemic irresponsibility* but rather a form of moral or practical failure with epistemic consequences. So the very feature that Axtell points to as a problem seems to be a good illustration of what's just right about my view.

2. One way to tell when normative categories are distinct: Plato, Firth, and Chisholm

I do not intend to engage in a debate about the history of epistemology. However, it does appear to me that from at least the *Meno* and *Theatetus* – which Chisholm interacts with[1] – the epistemic is that which provides the 'specific difference' (the species-defining characteristic) between mere true belief and knowledge. Chisholm seems to think (and I agree) that his theory of epistemic justification is a development of the notion of an 'account' introduced by Plato.

My position is that the Theatetic notion of (the core of) knowledge (with a nod to Gettier) as justified true belief places *epistemic justification* as the central concern of epistemology. Even if the aim is stated to be knowledge, truth is a free-rider. There is no epistemic merit in gaining the truth in an irrational manner, so the epistemic value of knowledge is supplied by rational element: justification.

Chisholm, in fact, seems to want to reduce normative categories as well, reducing the epistemic to the moral. That is, like me, he thinks the 'ethics of belief' really is just ethics (as does Zagzebski, as I point out in my "Reducing Responsibility: An Evidentialist Account of Epistemic Blame."[2]). It's just that, ironically, I think what he was calling ethics is in fact the core of epistemology, as I think Firth shows.[3] (Sometimes it seems that Locke has this in mind as well, for he speaks of a duty to God to use our faculties wisely.[4])

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[1] Roderick Chisholm, *Theory of Knowledge* (Englewood Cliffs: Prentice Hall, 1966), 5.

[2] *European Journal of Philosophy*, 2011, doi:10.1111/j.1468-0378.2010.00422.x.

[3] See Roderick Firth, "Ultimate Evidence," *The Journal of Philosophy* 53, 23 (1956), American Philosophical Association Eastern Division: Symposium Papers to be Presented at the Fifty-Third Annual Meeting, University of Pennsylvania, December 27-29, 1956 (Nov. 8, 1956): 732-739, "Chisholm and the Ethics of Belief," *The Philosophical Review* 68, 4 (1959): 493-506, and "Are Epistemic Concepts Reducible to Ethical Concepts," in *Values and Morals*, eds. A.

Firth-type cases show that the two notions can come apart. Here is a case I hope is sufficiently clear where one has all-things-considered moral reason to believe p yet all-things-considered epistemic reasons not to believe p. S has randomly sampled 15.58 trillion F's (that's one F for every dollar of US debt as of a few seconds ago). 99.99% of them have been G's. Every expert statistician on the planet agrees the sampling method was legitimate and has no worries about any features of the sample or the population which might prevent a standard inference. Let p be *Most F's are G's*. S has no other evidence pertaining to the proportion of G's among F's besides the sample distribution result. Clearly, S has an all-things-considered epistemic reason to believe p. But wait...S' has offered S $15.58 trillion (almost enough to pay off all US debt) to believe not-p. Let F and G represent properties which are of absolutely no practical importance. Perhaps at issue is the proportion of teenage boys scores on video games which have a '2' in them somewhere. There are no counter-offers or competing concerns. Clearly, S has an all-things-considered practical reason to (attempt to) bring it about that she comes to believe not-p. The reason we believe in these two distinct kinds of normativity is because we have two distinct kinds of reasons. This latter fact we know by contemplating clear cases like the exaggerated Firth case I have just provided (actually, I think we can know this by common sense, but the example confirms it). What no responsibilist has done to my knowledge is provide a case in which 'epistemic responsibility' is clearly distinct from other notions of normativity.

3. Why internalist evidentialism has no destructive practical consequences: disagreement and 'epistemic virtue'

Axtell raises two charges of negative consequences of internalist evidentialism. (N.B. 1: One can be an evidentialist without being an internalist, and there several versions of internalism. N.B. 2: That a thesis has adverse consequences is no evidence that it is false.) The first charge is original; the second charge is not (and is not intended to be). The first charge is that evidentialism cannot support reasonable disagreement (which he identifies with Rawlsian pluralism, which I

Goldman and J Kim (Dordrecht: D. Reidel, 1978), 215-229, and cf. Susan Haack, "The ethics of belief reconsidered," in *The philosophy of Roderick M. Chisholm*, ed. L. Hahn (LaSalle: Open Court, 1997), 129–144.

[4] "He that believes without having any reason for believing, may be in love with his own fancies; but neither seeks truth as he ought, nor pays the obedience due his Maker, who would have him use those discerning faculties he has given him." (John Locke, *Essay Concerning Human Understanding*, Book IV, Chap 15, Section 5).

doubt is correct). This charge is false, as I demonstrate in my forthcoming "Dealing with Disagreement from the First Person Perspective: A Probabilist Proposal."[5] In short, internalist evidentialism only calls for suspension of judgement in an idealized case: two people with credences symmetric about .5 with exactly the same evidence and exactly the same reliability. If any of these variables change, then my internalist evidentialst theory of epistemic peer disagreement entails that we shift our views with the consensus. And this seems like precisely the reasonable thing to do. So, far from having negative consequences in the realm of disagreement, a properly scientifically-minded, probabilistic internalist evidentialism can have quite salutary consequences in cases of disagreement.

The second negative consequence of internalist evidentialism Axtell mentions is best summarized by Alvin Goldman. "The main problem facing deontological evidentialism is to account for the virtues of evidence gathering."[6] This is simply false. Most people care about the truth. That is, they desire to have true beliefs (how exactly to state the relevant desire is actually a bit tricky). And, given this desire, dispositions toward effective evidence gathering will have (in cooperative circumstances) a tendency to produce true beliefs. That's a good-making feature for anyone who cares about truth. On the objective interpretation, it is a good for humans to have the truth. Therefore, there exists a practical reason to instantiate habits that promote the formation of true beliefs. On either of these accounts we ought (whether it is the subjective ought, the objective ought, or both) form those habits. This is all perfectly compatible with internalist evidentialism.

Again, the so-called epistemic virtues are just moral virtues with epistemic payoffs. The value of those virtues is wholly explained by this natural picture. There is nothing to be gained by calling these virtues 'epistemic.' It can only be misleading. For the forms of normativity involved are clear cases of practical rationality or teleology. It could be that not all dispositions which are beneficial for evidence gathering fit neatly into the category of moral virtue. I'm not committed to any thesis about natural kinds of virtues. There are lots of dispositions with lots of different kinds of effects, both good and bad. Furthermore, the kinds of traits which responsibilists name 'epistemic virtues,' e.g. conscientiousness, bear no necessary connection to success in evidence gathering. They may well have some intrinsic goodness derived from the goodness of the intentions which

[5] In *Disagreement and Skepticism*, ed. Diego Machuca, Routledge.

[6] Alvin I. Goldman, *Pathways to Knowledge: Private and Public* (New York: Oxford University Press, 2002), 56, as quoted by Axtell.

motivate such actions,[7] but it is wholly contingent which practices are *successful*. Methods helpful in one environment or world will be of ill effect in another.

But even if the charge were true, it would be irrelevant to the truth of evidentialism. Consider the parallel case to consequentialism. One sometimes hears the objection that the consequences of acts are too complicated to calculate, and so consequentialism offers no guidance in how to act. But consequentialism isn't *intended* to provide guidance in how to act. It is a *theory of right action*. It would be nice if that helped, and it would be unsurprising if it did, but it is no mark against the truth of a theory that it isn't *useful*, when it is only meant to be *accurate*.

4. That 'fallacies' are not always cognitive defects

Axtell alleges an "obvious inconsistency" in claiming that an agent is not subject to distinctively epistemic sanction when committing fallacies of reasoning or exhibiting cognitive biases. The problem with this suggestion is that, like most informal fallacies and cognitive biases, the ones he mentions are sometimes good modes of thinking and sometimes bad ones. (Obviously it is never good to *improperly* appeal to authority, but the adjective implicates that it sometimes is proper to appeal to authority, and it will sometimes be disputatious which is which.) In fact, that holds true for deductive 'fallacies' as well: scientific confirmation via successful prediction is a form of affirming the consequent.

But let's look at the two examples he gives.

> A. "I can't read or consider that recommended book on evolution
>
> because it will lead to ungodliness."
>
> B. "Others tell me not to read such rubbish, so rubbish it must be."

With respect to A, the belief expressed is either evidentially justified or it is not. If it is not, then that is part of what is wrong with it. And whether it is justified or not, the speaker either cares more about godliness than furthering their knowledge of the creation/evolution debate or they do not. If they do care more about godliness and they think there is a conflict between having that property and reading the books, then it seems perfectly appropriate not to read the books. It's a typical philosopher's mistake to think that one should always do more research. If the individual does not care more about godliness and, instead, cares more about the truth of the matter, then it is utterly imprudent not to read the books. 'Fallacy' doesn't even seem to be a helpful term here. The notion of a

[7] See Linda Zagzebski, *Virtues of the Mind* (Cambridge: Cambridge University Press, 1996) on this.

formal fallacy is comparably clear: to make an inference that is not truth-preserving. But the notion of an informal fallacy is, I assert, very fuzzy and unhelpful at the relevant level of generalization. It is sometimes said that to count as a fallacy at all – formal or informal – an item must be a part of an argument for the truth of some conclusion. It is not clear that A even qualifies as an inference in the relevant sense. It seems to be a clear case of deliberation. As such, it should be judged on a prudential basis.

As for B, whether this inference is good or bad depends on whether one's evidence supports the proposition that the 'others' involved are generally reliable. Of course, the *degree* to which it makes sense to check on the reliability of others depends on just one thing: how much is at stake. If there is not much at stake, then it makes perfect sense to just go with what people are saying, like if you step off of a train and want to know where to catch a cab and are in no hurry.

I see no relevant difference in this case between formal and informal fallacies. Suppose someone is attempting to prove that some wff the main operator of which is a negation is a theorem. They do their truth-tree and there are open branches and so conclude that it is not a theorem. The problem, suppose, is that the fact that a negation was the main operator of the wff caused them not to negate the wff before beginning the truth tree. This is a mistake in reasoning. Is this oversight *irresponsible*? Well, that depends on what is at stake. If the individual is a logic TA maybe they had a responsibility to be extra careful and maybe even work from a flowchart. But if not, if she's just passing the time waiting for the train, then there's no kind of blame *at all* that needs to be applied. It's a typical 'philosopher's mistake' to think that every act of bad reasoning is reprehensible.

5. On competence in achieving one's intellectual ends

Axtell insists upon more entanglement between the epistemic and the moral/practical than I have. But he never quite says how he envisions this entanglement. I have, in fact, presented a theory of the appropriate kind of entanglement. Epistemology defines a certain kind of value – epistemic value – which consists in realizing one's *telos* as a rational animal, a *reasons weighing* animal: having a degree of certainty which matches the weight of one's reasons (the sum total of which is one's evidence). When one has attempted to achieve this epistemic end and formed one's degree of belief, then one can consider what one desires and to what degree. What one ought to do in a case will be determined by instances of practical reasoning. Practical reasoning involves considering both one's desires and ones degrees of belief in the way regimented by decision theory.

It is in practical reasoning's use of degrees of belief that the epistemic and the moral/practical are properly entangled.

By "intellectual competence" Axtell seems to have in mind a certain set of skills or habits which include skill at identifying evidence. There is a bit of a generality problem here as Axtell doesn't flesh out what an exercise of this skill would look like. As far as I can tell, there is no set of practices at an appropriate level of generality that bear any necessary connection to identifying one's evidence, nor do I see such a connection between identifying one's evidence and forming true beliefs. If we take a 'thick' view of the so-called intellectual virtues – such as taken by Roberts and Wood[8] and Baehr[9] – then it is simply an empirical matter whether and when they are going to be beneficial. To take an example, sometimes being conscientious in research will lead to more evidentially justified beliefs and sometimes it will lead to less: maybe you stay up so late studying that you miss an important class or are over-tired and can't focus the next day. Identifying the best strategies for achieving our intellectual ends is best left to cognitive psychologists.

More broadly, Axtell appears to identify the epistemic with the truth-directed, but this is contentious. The western tradition of epistemology essentially starts on Plato's *Meno* and *Theatetus* in which Plato puzzles over what distinguishes – in nature and importance – knowledge from mere true belief. There is no epistemic merit merely in believing the truth. Epistemology is normative; it investigates how one *ought* to believe. But belief is not a normative notion nor is truth. The idea that truth or belief are normative rests on the metaphor that "belief aims at truth." There is a large literature on this, and I cannot get into it here, but my position is that it is dubious at best whether this metaphor latches onto anything true and of consequence to the present discussion. Here I can only say two things about my position.

First, I think it is *agents* who aim at belief (and goodness), not intentional states, whether beliefs or desires. Second, this does not indicate a lack of belief in natural teleology. I do think that beliefs and desires have functional roles and nondefectvieness conditions in an agent's mental economy to bring about rational actions. Do I think that a false belief is defective as such? No, I do not. Beliefs are either basic or inferred. An inferred belief that fits the non-basic evidence is everything it is meant by nature or Nature's God to be. It is insulated from any

[8] Robert Roberts Jay W. Wood, *Intellectual Virtues: An Essay in Regulative Epistemology* (New York: Oxford University Press, 2007).

[9] Jason Baehr, *The Inquiring Mind: On Intellectual Virtues and Virtue Epistemology* (New York: Oxford University Press, 2011).

further evaluation by the fact that the evidence stands between it and the world. I think it is perceptual states which are more likely to have some kind of accuracy condition, but there is nothing there for responsibilists to work with as far as I can tell. So goes it with basic beliefs. Their 'job' is to fit the empirical evidence. There is nothing more we can ask of them.[10]

Haack sums up the value of truth for humans aptly and is worth quoting at length.

> Intellectual integrity is instrumentally valuable, because, in the long run and on the whole, it advances inquiry; and successful inquiry is instrumentally valuable. Compared with other animals, we are not especially fleet or strong; our forte is a capacity to figure things out, hence to anticipate and avoid danger. Granted, this is by no means an unmixed blessing; the capacity that, as Hobbes puts it, enables men, unlike brutes, to engage in ratiocination, also enables men, unlike brutes, "to multiply one untruth by another." But who could doubt that our capacity to reason is of instrumental value to us humans?
>
> And intellectual integrity is morally valuable. This is suggested already by the way our vocabulary for the epistemic appraisal of character overlaps with our vocabulary for the moral appraisal of character: e.g., 'responsible,' 'negligent,' 'reckless,' 'courageous,' and, of course, 'honest.' And "He is a good man but intellectually dishonest" has, to my ear, the authentic ring of oxymoron.[11]

We philosophers tend to think that there is something important about believing the truth. If this is not just a prejudice (in which case there's just no question that moral and pragmatic norms are the only that can apply) but is rather an important human project, then there is a type of flourishing characteristic to such contact with reality. There seems something humanly defective with not caring whether one's beliefs are true or not. Plausibly, one is a *bad person* to the extent that they don't have a truth-oriented concern. But, whereas there is something *incoherent* about having degrees of certainty that do not match one's degree of evidential support, there doesn't seem to be an such analog in the case of having a justified belief which isn't true. This is a sign that we are dealing with two different kinds of normativity here.

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[10] For very different views, see Jonathan Kvanvig, "Truth is not the primary epistemic goal," in *Contemporary Debates in Epistemology*, eds. Matthias Steup and Ernest Sosa (Malden: Blackwell, 2005), 285-296 and Marian David, "Truth as the primary epistemic goal: a working hypothesis," in *Contemporary Debates in Epistemology*, 296-312. Though both views are at odds with mine, I think there are parts of each that support the kind of picture I'm painting here.

[11] Susan Haack, "Concern for Truth: What it Means, Why it Matters," in *The Flight from Science and Reason*, eds. Paul R. Gross, Norman Levitt, and Martin W. Lewis (New York: New York Academy of Sciences, 1996), 57-63.

It is well to keep in mind the three kinds of factors Jonathan Kvanvig urges us to attend to:

> we need to notice is that there are three dimensions here as well: the descriptive realm of how people go about making sense of things, the normative realm about how they should or should not do so, and the evaluative realm of how it is best done ... the perspectivality platitude that what is appropriate to think or do is a matter of one's total perspective on the world and one's place in it. The rest is a matter of things going well or badly, and ... the world is never as cooperative as we would like in lining up the good and the right.[12]

6. On diachronic considerations in belief

Assuming that we *do* or *morally should* have a desire for truth, the theory of inquiry is an important area of study. It can appeal to epistemology for a characterization of notions that will be important like evidence and justification. It will involve ethics in giving an account of how this duty is fulfilled. (My own view is, roughly, that one has responsibly inquired when further inquiry has no positive expected utility.) It will involve psycyhologists doing empirical research on just what modes of behavior in which conditions lead ones to better gather and assess evidence. The theory of inquiry is, then, an *interdisciplinary* field of study that, by definition, involves *multiple disciplines* (and sub-disciplines). A good model of this is so-called 'cognitive science,' which involves the cooperation of philosophy of mind, neurology, and psychology. As I have said before, there is more at stake than mere correct taxonomy. Aristotle pointed out long ago the importance of regulating our expectations and methods to the discipline. We stand only to gain confusion by misunderstanding what kind of theorizing we should be engaging in to understand the phenomenon in question. The theory of inquiry is too important to risk that confusion.

[12] Jonathan Kvanvig, "Epistemic Normativity," in *Epistemic Normativity*, eds. John Turri and Clayton Littlejohn (Oxford University Press, forthcoming).

STUMBLING IN NOZICK'S TRACKS

John TURRI

ABSTRACT: Rachael Briggs and Daniel Nolan have recently proposed an improved version of Nozick's tracking account of knowledge. I show that, despite its virtues, the new proposal suffers from three serious problems.

KEYWORDS: knowledge, tracking theory, sensitivity, Robert Nozick, Rachael Briggs, Daniel Nolan

Following a recent trend,[1] Rachael Briggs and Daniel Nolan seek to bolster Nozick's tracking theory of knowledge.[2] Nozick proposed that you know that P iff:

(1) P is true.

(2) You believe P.

(3) If P hadn't been true, then you wouldn't have believed P.

(4) If P had been true, then you would have believed P.

Nozick's view has been rejected over and over again in the literature,[3] almost invariably on the basis of purportedly devastating counterexamples.[4]

Briggs and Nolan propose an alternative analysis of knowledge that resists some of the counterexamples.[5] Their solution is to rely on *dispositions* rather than subjunctive conditionals in the third and fourth clauses of the analysis. They propose that you know that P iff:

(1) P is true.

(2) You believe P.

(3*) You are disposed to not believe P in the circumstances where P is not true.

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[1] E.g., Fred Adams and Murray Clarke, "Resurrecting the Tracking Theories," *Australasian Journal of Philosophy* 83, 2 (2005): 207–221; Sherrilyn Roush, *Tracking Truth: Knowledge, Evidence, and Science* (New York: Oxford University Press, 2006); Tim Black and Peter Murphy "In Defense of Sensitivity," *Synthese* 154, 1 (2007): 53–71.

[2] Rachael Briggs and Daniel Nolan "Mad, Bad and Dangerous to Know," *Analysis* 72, 2 (2012): 314–316.

[3] Robert Nozick, *Philosophical Explanations* (Cambridge: Harvard University Press, 1981).

[4] Jonathan Schaffer, "Perceptual Knowledge Derailed," *Philosophical studies,* 112, 1 (2003): 31–4; Jonathan Vogel, "Subjunctivitis," *Philosophical studies* 134, 1 (2007): 73–88; Saul Kripke, *Philosophical Troubles* (New York: Oxford University Press, 2011), ch. 7.

[5] Briggs and Nolan, "Mad, Bad."

(4*) You are disposed to believe P in the circumstances where P is true.[6]

Call this the *dispositional tracking account*, as opposed to Nozick's original *subjunctive tracking account*.

I agree that the dispositional tracking account improves on the subjunctive tracking account in some ways. It handles some cases that the subjunctive account can't, as Briggs and Nolan effectively argue. But the dispositional account still suffers from several very serious problems, which it shares in common with the subjunctive account. I will focus on three such remaining problems.

First, consider this case, which seems to be a straightforward counterexample to the dispositional tracking account:

> (DOOR) The automatic door improbably malfunctions and closes prematurely, striking Dora hard on her ankle. This causes excruciating pain, on which basis Dora believes that she is in pain. But very easily the door could have delivered a mere glancing blow, causing only very minor discomfort rather than pain. Moreover, Dora is a hypochondriac disposed to believe that she is in pain, even when she experiences only minor discomfort.

Dora knows that she is in pain. But the dispositional tracking account implies that she doesn't know, because she fails to satisfy condition 3*. Dora is not disposed to not believe that she is in pain in the circumstance where she isn't in pain.

Second, the dispositional tracking account entails that we can't know that we exist, which is absurd. I know that I exist. But I lack any disposition to do, or to not do, anything when I no longer exist. In particular, I am not disposed to not believe that I exist in the circumstance where I don't exist. So the dispositional tracking account implies that I don't know that I exist, because I fail to satisfy condition 3*. Of course, it is *true* that I will not believe that I exist in the circumstance where I don't exist. But the reason it is true isn't that I have a disposition to not believe that I exist in such circumstances.

Third, the dispositional tracking account makes it strangely difficult to have second-order knowledge. Take any mundane belief that satisfies the conditions of the dispositional tracking account, such as your belief that you ate a sandwich for lunch, and let 'Q' abbreviate this proposition you believe. Clearly you know that

[6] In their official formulation, instead of 'is not true' in 3*, Briggs and Nolan write 'does not obtain'; and instead of 'is true' in 4*, they write, 'obtains'. But it's much more natural to speak of a proposition *being true* (or not) than to speak of it *obtaining* (or not). And in explaining their view, Briggs and Nolan speak of propositions being true, rather than obtaining. For example, "In each case, our subjects have an epistemically relevant disposition to believe the relevant proposition if it is true, and not to believe the relevant proposition if it is not true." For these reasons, I use 'true' rather than 'obtains' in formulating their view.

Q, and the dispositional account gets this right. Now suppose that you also believe that your belief that Q is true, and let 'R' abbreviate this proposition you believe. R seems like something that you could easily know, in just about any case where you know that Q. But you fail to satisfy condition 3* with respect to R. That is, the following disposition-ascription is false, at least in cases where you know that Q:

> You are disposed to not believe R in the circumstances where R is not true.

If you are neither incoherent, peculiarly diffident, nor basing your belief that Q upon grounds that would appear inadequate upon reflection, then you are disposed believe that your belief that Q is true, even when it is in fact false. And, typically at least, if you know Q, then you are neither incoherent, peculiarly diffident, nor basing your belief upon reflectively inadequate grounds. (Of course, this disposition of yours is defeasible – you aren't irredeemably stubborn or incorrigible – but so are most dispositions.) Thus, according to the dispositional tracking account, you know that you ate a sandwich for lunch, but you don't know that you know that you ate a sandwich for lunch. I submit that this result is implausible. And the result will generalize to many of our first-order beliefs.

In sum, although the dispositional tracking account is an improvement over the original subjunctive tracking account, it still suffers from many of the same serious problems. Despite making modest progress along the path Nozick helped to pioneer, it has been halted in its tracks.[7]

[7] Thanks to Rachael Briggs and Angelo Turri for helpful conversation and feedback.

HISTORY OF EPISTEMOLOGY

FREGE ON IDENTITY. THE TRANSITION FROM *BEGRIFFSSCHRIFT* TO *ÜBER SINN UND BEDEUTUNG*[1]

Valentin Sorin COSTREIE

ABSTRACT: The goal of the paper is to offer an explanation why Frege has changed his *Begriffsschrift* account of identity to the one presented in *Über Sinn und Bedeutung*. The main claim of the paper is that in order to better understand Frege's motivation for the introduction of his distinction between sense and reference, which marks his change of views, one should place this change in its original setting, namely the broader framework of Frege's fundamental preoccupations with the foundations of arithmetic and logic. The Fregean thesis that mathematics is contentful, and its defense against *formalism* and *psychologism*, provides us an valuable interpretative key. Thus, Fregean senses are not just the mere outcome of some profound reflections on language, rather they play an important role in the articulation of Frege's program in the foundations of arithmetic

KEYWORDS: Fregean senses, informative identity, contentful mathematics

1. Introduction

Frege's account of identity is puzzling, and his views on this subject continue to occupy contemporary philosophical discussion. This paper aims to explain why and how Frege made the transition from his theory of identity proposed in *Begriffsschrift* (hereafter, *Bgs*)[2] to the one presented in *Über Sinn und Bedeutung* (hereafter, *SB*).[3] Recently, a series of papers[4] dedicated to this subject have

[1] This paper was made within *The Knowledge Based Society* Project supported by the Sectorial Operational Programme Human Resources Development (SOP HRD), financed by the European Social Fund, and by the Romanian Government under the contract no. POSDRU ID /89/1.5/S/56815.

[2] Gottlob Frege, *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (Halle: I. Nebert, 1879), translated in Gottlob Frege, *Conceptual Notations and Related Articles*, trans. and ed. Terrell Ward Bynum (Oxford: Oxford University Press, 1972), and selections in *The Frege Reader*, ed. Michael Beaney (Oxford: Blackwell, 1997).

[3] Gottlob Frege, "Über Sinn und Bedeutung," *Zeitschrift für Philosophie und philosophische Kritik* 100 (1892): 25-50, translated as "On Sense and reference" in *Translations from the*

appeared in the *Canadian Journal of Philosophy*, but unfortunately none of them explains conclusively Frege's motivation for this change. After presenting some recent contributions to this debate, I will focus on what I think was Frege's motivation for changing his views on identity. The main claim of the paper is that, in order to better understand Frege's motivation for the introduction of his distinction between sense and reference, we should seriously consider its original setting, namely the broader framework of Frege's fundamental preoccupations with the foundations of arithmetic and logic. The 'standard interpretation' is basically the narrow interpretation which holds that in *SB* Frege criticizes and rejects the account of identity of *Bgs*. The standard interpretation considers Frege's change of view only within the framework of philosophy of language, and assesses his theory of meaning solely from this perspective. In contrast with this point of view, I advocate an interpretation which considers his views on identity in the wider context on mathematics and logic.

Mike Thau and Ben Caplan[5] attacked the 'standard interpretation' and held that Frege never gave up his *Begriffsschrift* account of identity. I believe that their interpretation is mistaken, and I think that Richard Heck[6] has refuted this position conclusively. My goal here is to show why Frege came up with a new view of identity, thus completing Heck's refutation of this attack on the standard interpretation. What is wrong with the standard view is not that it claims that Frege changed his position concerning identity, but its failure to consider the rationale underlying this change. Based on the traditional way in which one has commonly learned that philosophy of language and philosophy of mathematics are disconnected philosophical fields, our natural inclination is to judge things separately; this approach is mistaken, and we shall shortly see why.

A recent affirmation of Thau and Caplan's claim that in *SB* Frege did not reject his earlier view of identity in *Bgs* may be found in Bar-Elli.[7] Basically, Bar-

Philosophical Writings of Gottlob Frege, eds. Peter Geach and Max Black (Oxford: Basil Blackwell, 1960), and in *The Frege Reader*.

[4] Mike Thau, Ben Caplan, "What's Puzzling Gottlob Frege?" *Canadian Journal of Philosophy* 31, 2 (2001): 159-200; Richard G. Heck, "Frege on Identity and Identity-Statements: A Reply to Thau and Caplan", *Canadian Journal of Philosophy* 33, 1 (2003): 83-102; Gilead Bar-Elli, "Identity in Frege's *Begriffsschrift*: Where Both Thau-Caplan and Heck Are Wrong," *Canadian Journal of Philosophy* 36, 3 (2006): 335-370; Imogen Dickie, "Informative Identities in the *Begriffsshrift* and 'On Sense and Reference,'" *Canadian Journal of Philosophy* 38, 2 (2008): 269-288.

[5] Thau, Caplan, "What's Puzzling."

[6] Heck, "Frege on Identity and Identity-Statements."

[7] Bar-Elli, "Identity in Frege's *Begriffsschrift*," 357.

Elli's main claim is that in *Bgs* "Frege distinguishes there between names (*Namen*) and signs (*Zeichen*). The distinction is not explicitly stated, but it is used almost consistently in section 8. (…) A sign, in *Bs*,[8] just denotes its content; this exhausts its meaning. A name, in contrast, includes a mode of determination (*Bestimmungsweise*) of its content." Bar-Elli holds that we should distinguish a 'thin' semantics, in which signs refer directly to their contents, and a 'thick' semantics, in which names refer to their referents through a sense or a mode of determination. Thus, for Bar-Elli, the transition from *Bgs* to *SB* is the transition from the coexistence of a semantics of signs and a semantics of names in *Bgs* to the unified thick semantics of *SB*.

At least one issue is problematic here: the allegation that Frege distinguished between signs and names. Here, I deal only with the former point. Regarding the distinction between signs and names in *Bgs*, at least two points should be noted: First, in §1 Frege makes a distinction in the realm of signs between variables and constants. But what are names if not constants? So, at most, we can say that names are a subclass of signs. Second, in §8, Frege presents a geometrical example in which the apparently different points A and B are in fact one and the same, the difference between them consisting in the way in which they are determined. And it is true that in connection with these different 'modes of determination,' *A* and *B* are also called 'names,' but, at the very end of the section, Frege says explicitly:

> Now let
>
> ⊢(A ≡ B)
>
> mean that the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely.

Since Frege does not distinguish here or elsewhere between names and signs, the distinction between a thin and a thick semantics seems to be an unsustainable interpretation of Frege's semantics.[9] Moreover, what Bar-Elli calls a 'thin semantics' is not part of the Fregean view of how signs/names refer. Frege says explicitly in *Bgs* §8 that "one point is determined in two ways: (1) immediately through intuition and (2) as a point B associated with the ray perpendicular to the diameter." Thus this so-called 'thin semantics' (basically, a Millian view of proper names) is seen as a case of a thick semantics: to be determined directly in intuition is, for Frege, just another way of being

[8] Bar-Elli's shortcut for *Bgs*.

[9] As we will see shortly, for Frege mathematical signs, including mathematical names, have content in the sense that they do refer to objects.

determined. Moreover, since the difference between the thin and thick semantics is based on the alleged difference between signs and names, it is necessary to understand why Frege held this in *Bgs* but dropped it later. No explanation is offered by Bar-Elli.

In another recent paper dedicated to this topic, Imogen Dickie[10] explains the transition between Frege's account of the informativeness of identity statements from *Bgs* to *SB* in terms of a difference between two senses of 'informative.' Dickie holds that in *Bgs* Frege is concerned with 'evolutionary informativeness': the transition, from the fact that a subject may understand two co-referential names without knowing that they co-refer, to the situation when the subject finds that they do co-refer, constitutes an epistemic advance. Dickie holds that in *SB* Frege is concerned with 'rational informativeness': the substitution of co-referring expressions in a proof preserves truth, but may transform a logically self-evident chain of inferences into one which is not, or vice-versa. This distinction is subtle and interesting, yet Dickie doesn't offer much textual evidence to show that these different senses of informativeness are separately connected with *Bgs* and *SB*. There are three questions to be addressed here. First, why cannot that which is informative in the evolutionary sense also be so in the rational sense, and conversely? This problem demands attention since the *SB* theory of identity is assumed to be an advance on the *Bgs* account of identity and so is supposed to provide something over and above what *Bgs* explains, with the addition of better explanations of new facts. But if they are different, then it follows that the new theory of identity of *SB* cannot cope with evolutionary informativeness; and I do not think that this is the case. Second, why should we hold that in *Bgs* Frege is concerned only with evolutionary informativeness, since the aim of *Bgs* is precisely to secure mathematical proof? Dickie does not provide any argument in this regard. Third, why should we consider Frege in *SB* to be concerned with rational informativeness alone, since from the start he formulates the whole discussion in epistemic terms regarding mathematical knowledge, aprioriticity and cognitive value (*Erkenntniswert*)? Here again, Dickie does not address this issue. So, although Dickie's analysis of informativeness seems very interesting and promising in the overall context of Frege's works, it is still difficult to understand why Frege has changed his views on identity.

Another series of papers devoted to this subject[11] approaches the problem from a different angle. Both Robert May and Richard Heck try to understand and

[10] Dickie, "Informative Identities."

[11] Richard G. Heck, "The Julius Caesar Objection," *Language, Thought, and Logic: Essays in Honour of Michael Dummett*, ed. Richard G. Heck (Oxford: Oxford University Press,1997),

explain Frege's account of identity and, more generally, his semantics from the perspective of his work in the foundations of mathematics. I am sympathetic with their approach, yet I think that they err in what they think was Frege's mathematical setting responsible for this change. Basically, their main claim is that Frege's logicist thesis is in fact responsible for the introduction of the sense/ reference distinction. As we'll see shortly, this is only partially true.

2. Identity

Logically speaking, we can distinguish two notions of identity. One is *numerical identity* or identity proper, and states that if "a is identical with b," then, in fact, 'a' and 'b' are just different names for the same object; a and b are the same under all aspects. However, we may use a second notion of identity: a and b are 'identical' only under one or some aspects, but not all. "Peter had an accident and his car was totally destroyed. But he went out and bought the same car" means that he bought the same model of car, but not the numerically identical one. The latter is sometimes called *qualitative identity*, whereas the former, by contrast is *quantitative identity*.[12]

This distinction is acknowledged in *Bgs* by the use of two different signs. '=', as in "3 x 7 = 21," means mathematical equality, whereas the second is '≡', and is defined in §8 as 'identity of content.' It is unlikely that the introduction of the latter is just a regrettable lack of rigor, since the main purpose of *Bgs* is to provide an *exact language* suitable for doing exact science. Thus, we have a formal sign for mathematical equality (=), which, from a logical point of view, is just a variant of qualitative identity, and a sign for 'identity of content' (≡), which is numerical identity. Yet, in the domain of numbers, the difference between them vanishes and later Frege will drop this notation and acknowledge that in mathematics

273-308; Robert May, "Frege on Identity Statements," in *Semantic Interfaces: Reference, Anaphora, and Aspect*, eds. Carlo Ceccheto, Gennaro Chierchia, and Maria Teresa Guasti (Stanford: CSLI Publications, 2001), 1-61; Richard G. Heck, "Julius Caesar and Basic Law V," *Dialectica* 59, 2 (2005): 161-178; Richard G. Heck,"Frege and Semantics," in *Essays on Frege's Conception of Truth*, ed. Dirk Greimann, *Grazer Philosophischen Studien* 75 (2007): 27-63; Richard G. Heck, Robert May, "Frege's Contribution to Philosophy of Language," in *The Oxford Handbook of Philosophy of Language*, eds. Ernest Lepore and Barry C. Smith (Oxford: Oxford University Press, 2006), 3-39.

[12] Note that in *Methods of Calculation based on an Extension of the Concept of Quantity* Frege calls 'quantitative identity,' what here is called 'qualitative identity.' However, as Frege himself will later acknowledge, in mathematics, in the pure quantitative domain, we should regard equality (identity under the quality of quantity) as numerical identity or identity proper. I shall discuss this in detail very shortly.

equality must be interpreted as numerical identity. Already at the time of *Grundlagen*[13] (hereafter, *Gl*), he seemed to realize this, but it emerges explicitly in *Grundgesetze*[14] *(*hereafter, *Gg*):

> The primitive signs used in my *Begriffsshrift*, are to be found again here with one exception. Instead of the three parallel lines I have preferred the ordinary sign of equality [*Gleichheit*], since I have convinced myself that it has in arithmetic precisely the *Bedeutung* that I wish to designate [*bezeichnen*]. I use, that is, the word 'equal' ['*gleich*'] with the same *Bedeutung* as 'coincident with' ['*zusammenfallend mit*'] or 'identical with' ['*identisch mit*'], and this is also how the sign of equality is actually used in arithmetic. The objection that might be raised to this will probably rest on an inadequate distinction between sign [*Zeichnen*] and what is designated [*Bezeichnetem*]. Admittedly, in the equation '$2^2 = 2 + 2$' the left-hand sign is different from the right-hand sign; but both designate [*bezeichnen*] or refer to [*bedeuten*] the same number.

However, this issue raises the following question: why didn't Frege use numerical identity from the very beginning? What prevented him from thinking of '≡' as '='? One possible answer may concern the general purpose of *Bgs*: as a language suitable for general science, a *begriffsschrift*[15] is not limited to mathematics and should somehow capture both notions of identity. For example, in physics or chemistry, one oxygen atom is identical with another, but this only means that they are qualitatively identical as atoms; as objects they are distinct and numerically different. However, this applies to spatio-temporal objects. Mathematical objects as numbers are for Frege logical objects, and are thus not constrained by any spatio-temporal limitation. Thus, in the domain of numbers equality may be seen as identity proper. The following passage from *On the Concept of Number* is very suggestive:

> I cannot repeat the substance of my *Grundlagen* here. (…) There are various designations for any one number. It is the same number which is designated by "1+1" and '2'. Nothing can be asserted of 2 which cannot also be asserted of 1+1; where there appears to be an exception, the explanation is that the signs '2' and "1+1" are being discussed and not their content. It is inevitable that various signs

[13] Gottlob Frege, *Die Grundlagen der Arithmetik, eine logisch-mathematische Untersuchung über den Begriff der Zahl* (Breslau: W. Koebner, 1884) translated as Gottlob Frege, *The Foundations of Arithmetic*, trans. J.L. Austin, 2nd ed. (Oxford: Blackwell, 1953).

[14] Gottlob Frege, *Grundgesetze der Arithmetik*, 2 vols. (Hildesheim: Olms, 1962); preface, introduction and sections 1-52 of vol. I and appendix to vol. II translated in Gottlob Frege, *The Basic Laws of Arithmetic: Exposition of the System*, ed. Montgomery Furth (Los Angeles: University of California Press, 1964); this quote is from *The Frege Reader*, 197.

[15] By the upper case italics '*Begriffsschrift'* or *Bgs*, I refer to Frege's well-known work, whereas the lowercase 'begriffsschrift *'*stands for the formal language presented in *Bgs* and *Gg*.

> should be used for the same thing, since there are different possible ways of arriving at it, and then we first have to ascertain that it really is the same thing we have reached. 2 = 1+1 does not mean that the contents of '2' and "1+1" agree in one respect, though they are otherwise different; for what is the special property in which they are supposed to be alike? Is it in respect of number? But two is a number through and through and nothing else but a number. This agreement with respect to number is therefore the same here as complete coincidence, identity. What a wilderness of numbers there would be if we were to regard 2, 1+1, 3 – 1, etc., all as different numbers which agree only in one property. The chaos would be even greater if we were to recognize many noughts, ones, twos, and so on. Every whole number would have infinitely many factors, every equation infinitely many solutions, even if all these were equal to one another. In that event we should, of course, be compelled by the nature of the case to regard all these solutions that are equal to one another as one and the same solution. Thus the equals sign in arithmetic expresses complete coincidence, identity.[16]

However, in this context, another problem arises, and it may be seen as a second possible answer to the question of why equality and identity are different in *Bg*. This is the problem concerning the content of mathematics, which is a very serious problem for Frege. The formalists may reply that if mathematical equalities express logical identities, then all mathematics collapses into assertions such as a = a. But this would be unacceptable for Frege, given his firm conviction that mathematics has an objective content and is not a mere game with signs. The formalist could argument runs as follows: let us assume that mathematics is contentful, and that there is a difference between sign and thing signified, so that mathematics is not about signs but about the objects they signify. In this case, mathematical equalities state identities among numbers as the objects signified by mathematical signs. But then, if mathematical equalities are true numerical identities, all mathematics collapses to the logically uninformative principle of identity. So, in what sense is mathematics contentful, when everything reduces to the contentless "a = a"? This is a serious objection, which must be addressed. Mathematical signs stand for mathematical objects and reference to such objects gives content to mathematics. But how can Frege hold these two apparently incompatible positions? Certainly, as Frege himself acknowledges, this is possible only with the help of his distinction between sense and reference, for one can now reply that mathematical equalities are true numerical identities which state relations within the realm of reference, yet hold that they are informative,

[16] Gottlob Frege, *Posthumous Writings*, trans. Peter Long and Roger White (Oxford: Blackwell, 1979): 85-6.

because the terms of the identity have different senses, and this marks their difference in cognitive value.

> The knowledge that the Evening Star is the same as the Morning Star is of far greater value than a mere application of the proposition "a = a" – it is no mere result of a conceptual necessity. The explanation lies in the fact that the senses of signs or words (Evening Star, Morning Star) with the same *Bedeutung* can be different, and that it is precisely the sense of the proposition – besides its *Bedeutung*, its truth-value – that determines its cognitive value. [17,18]

In sum, the *Bgs* view of identity has been modified not because of its alleged incapacity to deal satisfactorily with the problems generated by Millian views on proper names – as is commonly held in the literature surrounding this topic – but because of its role in the elaboration of Frege's contentful mathematics thesis.[19] In a footnote of *Gl* (§91) Frege says explicitly that the *begriffsschrift* is "designed, however, to be capable of expressing not only the logical form, like Boole's notation, but also the content of a proposition." The *content* – and this is consistent with its further splitting into sense and reference – is then seen as substantial information about the world, information that is 'carried' in the course of inference. So, a *begriffsschrift* has a dual role: to prevent the infiltration of subjective elements into the deductive chain of any scientific endeavor, and to carry information (about the world), information that is encapsulated in the structure of mathematical statements. Mathematical statements appear usually in the form of equations, equations constructed with the help of the equality sign '=', hence the importance of identity. Mathematical statements involve signs which designate numbers, and hence *the importance of a clear account about the*

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[17] The passage continues with a three-point characterization which obviously applies to Frege's account as well: "It follows from Dedekind's quoted remark that for him numbers are not signs, but the *Bedeutungen* of signs. These three points:

- the sharp distinction between sign and its *Bedeutung*,
- the definition of the equality sign as the identity sign,
- the conception of numbers as the *Bedeutungen* of number signs, not as the signs themselves,

hang most closely together and place Dedekind's view in the starkest contrast to every formalist theory, which regards signs or figures as the real objects of arithmetic" (*Gg*, II, §138; in *The Frege Reader* , 271).

[18] *Gg*, II, §138; in *The Frege Reader*, 271

[19] This thesis is given by Frege's strong claim that mathematics (contrary to *formalism* has content, and that this content (contrary to *psychologism*) is objective. 'Formalism' represents the position which claim that mathematics is nothing more than a mere game with empty signs, whereas 'psychologism' should be read as the claim that mathematical statements have and irreducible subjective content.

mechanism of denotation in the case of proper names.[20] Note that almost all I have said about identity so far is nicely expressed by Frege himself in a letter to Peano:

> [T]his does not yet explain how it is possible that identity should have a higher cognitive value than a mere instance of the principle of identity. […]
> *At this point my distinction between sense and meaning comes in in an illuminating way*. […]
> So nothing stands in the way of my using the equals sign as a sign of identity.[21]

It is clear that Frege's distinction between sense and reference plays an *illuminating* role in showing, against (the counterattack of) the formalists, that in mathematics we should take equalities as identities; yet mathematical statements are *contentful* and not "boring instances of this boring principle [of identity]." Moreover, this passage also indicates the context in which we should understand Frege's concern about the puzzling nature of identity statements. Accordingly, it is important to have a unique and clear understanding of equality in mathematics as identity; for equality is a central concept, and if we hold that mathematics has content, then of course this content should be correctly displayed by mathematical equations.[22] Thus, the moral of the story so far is that the motivation to show that identities like $a = b$ are informative and not just simple reiterations of the principle of identity is given by Frege's intention to establish in opposition with the formalists, that mathematical equations are capable of being substantive identities which enlarge our knowledge.

3. Identity and the sense-reference distinction

As we have seen, for Frege, mathematical equality is numerical identity, and two reasons seem to justify this step. The first is that it allows for a greater degree of unification and coherence in mathematics. The second concerns his struggle against the formalists: since mathematical statements are about objects, and since arithmetic includes equations, mathematical equality should therefore be taken to express a relation of identity between objects. But logical identity is just such a

[20] *Proper names* should be taken here in a broad sense which includes all singular terms.

[21] Gottlob Frege, *Philosophical and Mathematical Correspondence*, trans. Hans Kaal (Oxford: Blackwell, 1980): 126-8; my italics.

[22] *Gg*, vol II, §58, note A: "If mathematicians have divergent opinions about equality, this means nothing less than that mathematicians disagree as to the content of their science; and if we regard science as essentially consisting of thoughts, not of words and symbols, it means that there is no united science of mathematics at all – that mathematicians just do not understand one another. For almost all arithmetical propositions, and many geometrical ones, depend for their sense, directly or indirectly, upon the sense of the word 'equals'." In *The Frege Reader*, 261.

relation. Of course, now he has to counter a further attack of the formalists, namely that if mathematical equality is numerical identity, then most of mathematics collapses to mere instances of the cognitively uninteresting principle of identity. But for Frege, even though an equation states that we have an identity of *Bedeuntungen*, their *Sinne* are different and so we can see why they are not 'boring' identities. All this is made explicit by Frege in the previously cited letter to Peano in which he affirms explicitly that his S/R distinction "comes in in an illuminating way" in explaining how "it is possible that identity should have a higher cognitive value than a mere instance of the principle of identity."

It is clear now that the S/R distinction plays an important role in the 'unified' theory of identity, which is in fact more than just a mere unification of symbolization on new semantic grounds; rather it expresses the view that equations are about objects and are often informative identities. Thus, I find quite problematic the following characterization of Heck & May[23]:

> Though its application to identity-statements is extremely significant, it's important to observe that the distinction between sense and reference does not emerge from any particular concern with identity-statements. At the time of *Begriffsschrift*, Frege treated mathematical equality as a notion distinct from 'identity of content,' the latter being the notion governed by Leibniz's Law. But Frege must quickly have realized that the view is incompatible with a central tenet of logicism, namely, that there are no arithmetical notions with irreducibly mathematical content.

It is true that Frege was concerned with identity statements in mathematics. But his concern was to address the formalist criticism that if mathematical equalities are taken to be objectual identities, mathematical equations are no more informative then the principle of identity. But equality in *Bgs* is qualitative identity – not an 'irreducible mathematical notion,' but a special kind of logical identity. Having two kinds of identities at the time of *Bgs* wasn't a threat to logicism since both could be seen as 'logical' and thus arithmetic is still logic. Frege adopted the 'objectual identity view' in order to cope with the requirement of his *contentful mathematics thesis*, namely that arithmetic has objective content and thus is about logical objects. Therefore, (arithmetical) identities should be informative and so they should be more substantive than the mere principle of identity. Thus, a = b tells us more than a = a, and this 'tells us more' is nicely explained by Frege with the help of his S/R distinction.

> [I]dentities are, of all forms of proposition, the most typical of arithmetic. It is no objection to this account that the word 'four' contains nothing about Jupiter or

[23] Heck, May, "Frege's Contribution to Philosophy of Language," 22.

> moons. No more is there in the name 'Columbus' anything about discovery or about America, yet for all that it is the same man that we call Columbus and the discoverer of America. (*GI,* §57)[24]

It is interesting that "Columbus is Columbus" and "Columbus is the discoverer of America" express different thoughts, and thus have different cognitive values. Moreover, this means also that the senses of the two expressions, 'Columbus' and 'the discoverer of America,' are different, which certainly contrasts with the common interpretation of Frege as a strict descriptivist. Also, it may be objected that since this is a paragraph from *GI*, and at that time Frege did not draw his S/R distinction, it may be somehow anachronistic to judge the issue in these terms. However, on the one hand it is clear that his views were basically the same, and the acknowledgement of his famous distinction was just a semantic refinement of the same view about the world. On the other hand, the following passage shows explicitly that, after the distinction has been introduced, he thought of things in exactly the same terms:

> So the two signs are not equivalent from the point of view of the thought expressed, although they designate the very same number. Hence I say that the signs '5' and '2+3' do indeed designate the same thing, but do not express the same *sense*. In the same way 'Copernicus' and 'the author of heliocentric view of the planetary system' designate the same man, but have different senses; for the sentence "Copernicus is Copernicus" and "Copernicus is the author of heliocentric view of the planetary system" do not express the same thought.[25]

So, mathematical equations and logical definitions are grounded on identities, and thus they express the fact that on either side of the equation we have different names for the same object. But do we have an identity of sense as well? That, certainly, would make all statements involving identity analytic statements, but in this way we cannot explain the fact that mathematics is contentful.

Summing up, the focus of this paper has been on a new interpretative perspective: Frege's original contributions, especially those on language and semantics, have been viewed from the perspective of his philosophy of mathematics. My claim is that the fact that *mathematics is contentful* is the true key to a better understanding of Frege's insights and results. From this perspective the connection between Frege's views on language and mathematics are seen as an organic whole, and so the role of Fregean senses in his overall project becomes clear. Frege's accounts of identity arose in the context of his struggle against formalism and psychologism, and thus it should be clear now that he introduced

[24] In Frege, *The Foundations of Arithmetic*, 69.

[25] In Frege, *Posthumous Writings*, 225.

the S/R distinction in the framework of securing the contentful mathematics thesis. Thus, Fregean senses are not just the outcome of a mere linguistic analysis, rather they play an important role in the articulation of Frege's program in the foundations of arithmetic.

EPISTEMOLOGICAL PERSPECTIVES IN LATE ANTIQUITY – A DIALOG BETWEEN RATIONALISM AND EMPIRICISM IN THE SCIENTIFIC WRITINGS[1]

Adrian MURARU

ABSTRACT: Given the particular character of Ancient Literature, I considered it useful to approach the issue from the perspective of the Philosophy of Science: Epistemology and the Philosophy of Science stem from the same source, and this aspect is all the more patent for Antiquity. In fact, the two perspectives that I mentioned in the subtitle, Empiricism and Rationalism, both represent epistemological choices and approaches specific to the Sciences, as well as to the Philosophy of Science, in the manner that they were practiced in Antiquity. This present study argues that Empiricism noticeably distinguishes itself from Rationalism, not merely in the philosophical works of the above-mentioned period, but also in its non-philosophical literature, especially the one pertaining to Science. Consequently, this study aims to indicate the major lines of thought in the Ancient Philosophy of Science, which reflect themselves in Epistemology in an unmediated manner.

KEYWORDS: Late Antiquity's epistemology, philosophy of science, Galen, rationalists, empiricists

1. Sources

As one might rightly presume, the primary sources for this research consist of documents composed at that time, and successfully preserved onto this very day. They pertain to various disciplines, from Religion to Science, from Theology to sheer Literature. In my previous papers on the subject,[2] I have tried to at least partially indicate how the epistemological endeavors of Late Antiquity were structured, and I mainly focused on the science of the period. Nevertheless, as

[1] This paper was elaborated within the POSDRU/89/1.5/S/56815 project Knowledge – Based Society – research, debates, perspectives, co-financed by the European Union and the Romanian Government from the Social European Fund, through the Operational Sector Program for the Development of Human Resources 2007-2013.

[2] See Adrian Muraru, "Philosophy of Science and Epistemology in the Scientific Writings of Late Antiquity," in *Significance and Interpretation within the Knowledge Based Society* (Iaşi: Institutul European, forthcoming), Adrian Muraru, "Some Epistemological Perspectives in the First Christian Centuries," in *Philosophy and the Knowledge Based Society*, ed. Adrian Muraru (Iaşi: Institutul European, 2012), 21-30.

anyone should expect, research is significantly impeded by the state of the source-texts (very few have survived onto the current day), and also by the type of censoring filter applied, over time, to their transmission through copying. By this, I do not just mean ideological censorship, but also one of a subtler kind. It stems from slanted intellectual interests that both medieval and modern readers harbored, so that they favored particular works/authors at the fatal expense of others. An additional difficulty arises from the situation of (potentially) relevant sources: many of them fell out of grace from contemporary research interests, and hence, they do not enjoy the privilege of appropriately accurate (critical) editions.

2. Topic

On the other hand, the topic of the Philosophy of Science is of secondary interest for contemporary scholarship on Late Antiquity, as far as non-philosophic works are concerned. There are multiple reasons for this aspect: from placing en excessive emphasis on philosophic literature, which is considered the most relevant, to the preeminence of specific topics in contemporary research. All these aspects describe the generic situation of the field: scholarly literature is highly selective, so that the Philosophy of Science (and the underlying Epistemology) remains rather marginal for the interests of modern research. Most studies/books end up discussing the Epistemology of Late Antiquity only in its secondary sense, by referring to the period's various theories of knowledge, and focusing mostly on the authors that are philosophically relevant. Thus, one may still encounter studies on the Epistemology of Plotinus or on that of Aristotelian commentators from Late Antiquity, but only at the outskirts of scholarly literature, in a marginal segment, because the ontological or metaphysical topics dominate the field.

3. Subject

Nevertheless, an epistemological interest persists not just in the philosophic works, but also in the scientific ones. It stems from certain conceptual options within the Philosophy of Science: the main argumentative choices are exercised in the field of Science, as well, and this research path is less traveled. This study operates with the working hypothesis that there are fully cogent and coherent epistemological doctrines underlying the main argumentative choices in the Philosophy of Science. Henceforth, I will seek to flesh out these notions from the scientific works of Antiquity. The hallmark of the period is that very few of its scientific writings were preserved. For this reason, the method I chose for this study has been to carefully follow the writings of a second-century AD doctor (but also philosopher), namely Galen, and then interpret – through the conceptual lens

that they provide – the surviving evidence on various argumentative choices from the Philosophy of Science and, implicitly, from Epistemology. I chose to focus on the work of Galen because of its considerable size, which renders it as a more significant sample for the flavor of the intellectual milieu in Late Antiquity. In turn, this makes it easier to observe and highlight trends in the doctrines specific to the Philosophy of Science. Fortunately, most of Galen's works survive to this day: their medical content made them practically useful to posterity, so that they were copied, and thus, preserved.[3] The fact that they were saved *only* due to their *practical* utility is suggested by the fate that the same author's philosophic and philological works suffered – the most important ones disappeared, almost without a trace. Galen, whose work served as the main source for this study, is also considered a philosopher by his contemporaries (such as Alexander of Aphrodisia).[4] One can only regret that the corpus of his work is available nowadays only with significant lacunae. Galen transmits essential information about the three medical schools of thought in his time, and beyond his accounts, we may only be content with fragments from authors belonging to these schools (in *Die griechische Empirikerschule,* Deichgraeber collected fragments relevant to the Empiricist stream of thought, while Manuela Tecuşan has edited the first volume in a series that seeks to collect all the surviving fragments from the Methodists: *The Fragments of the Methodists. Volume One: Methodism Outside Soranus*).

Therefore, Galen's writings provide testimonies on the different epistemological attitudes from authors in Late Antiquity. He, himself, adopted a well-defined epistemological perspective, and since we lack other relevant written evidence on the debates from this period, our only solution is to appeal to the single available 'witness.' The author often discusses the fundamental differences between the main medical schools of the period, the empirical and the Rationalist one. He also

[3] One should remark that the entire medical literature that preceded Galen has disappeared, because the second-century AD doctor-philosopher was considered normative. There are few exceptions: *The Hippocratic Corpus* (whose survival can be explained by its professional prestige, authoritatively confirmed by Galen), *The Gynecology* of Soranus (incidentally, also praised by Galen), or the *Materia medica* of Dioscorides.

[4] The correlation between Medicine and Philosophy is obvious from the very first lines of the treatise *An Outline of Empiricism*: „All doctors who are followers of experience, just like the philosophers who are called Skeptics, refuse to be called after a man, but rather want to be known by their frame of minds" (Galen, *Three treatises on the nature of science*, trans. M. Frede (Hackett: Indianapolis, 1985). Toward the end of the treatise, Galen returns to the comparison between the Empiricists and the Skeptics, insisting on the similitude of their attitudes (see Ch. 11). One should remark how the Empiricists use Democritus (see *On Medical Experience*, 9, where the atomist is cited: "experience and difficulties have taught people to do these things").

mentions the 'methodic' school, a medical 'sect' that was trying to suggest a different epistemological approach. Three of Galen's writings are essential for understanding the differences between the various medical schools.[5] The first one is *On Sects – for Beginners*, a medical treatise that focuses on the 'empirical' sect, and which was preserved in Greek. The same topic dominates the treatise titled *An Outline of Empiricism*, preserved in Latin (in the translation of Nicolaus de Regio). Finally, the third paper, which only survived in Arabic[6] (just a few fragments were preserved in Greek), is entitled *On Medical Experience*.[7] For this final work, because of the lacking necessary knowledge in Arabic, I will use the English translation by R. Walzer. The linguistic diversity of these surviving works suggests a lot about the fate of Galen's writings, but also underlines to what extent prudence should accompany research.

Unfortunately, Galen is the only author who transmits any information on the medical sects mentioned above: one should note that the terms 'Empiricist' and 'Rationalist'[8] (which enjoy a glorious history in European Philosophy) seem to have been first used precisely on the occasion of these debates, which did not strictly pertain to the medical field. As such, one should be mindful that Galen's view may skew the information (for Galen, too, seems to have had his own firm perspective on Medical Epistemology), even if he himself is aware of this aspect, and transparently admits it. In one of the writings of his old age, when he tries to set order among the works that he had authored, Galen notifies the reader of his own potential bias. In *On My Own Books*, he professes that he had always refused to uncritically adhere to the opinions of any particular medical sect, and that he had always sought to elaborate his own intellectual stance, from which he could critically scrutinize other authors' opinions with detachment (1). Nevertheless, this type of 'eclecticism' does not spare Galen from an inherent bias. Moreover,

[5] Scholars dispute the authorship of a different writing, which offers abundant information on the medical schools of Late Antiquity. This work is usually attributed to Galen (*On the Best Sect*), but because of the reason I explained above, I will avoid discussing it in this paper.

[6] In fact, as R. Walzer, the editor himself, attests (pp. VI-VII), the Arabic version in turn follows the lost Syriac translation. Hence, the history of this treatise's transmission follows a far more winding course, from Greek, into Syriac, and finally into Arabic.

[7] For *De sectis ingredientibus*, I used the G. Helmreich edition (*Galeni Pergameni opera minora*, vol. III, Leipzig, 1893), for *Subfiguratio empirica*, I used the K. Deichgraeber edition (*Die griechische Empirikerschule*, Berlin, 1930, reedited in 1965), and for *On Medical Experience*, I used the R. Walzer edition (Oxford, 1944).

[8] Galen recounts that these medical schools (*On Sects – for Beginners*, 1) had also received other names. The Empiricists were also called "observationalists," while the Rationalists were also called "dogmatics," or "analogists."

the author's often impetuous writing style should serve as a warning to his audience, as he frequently slips from sober expositions of other authors' works to imprecations against them. And yet, several elements prove advantageous to the inquiry at hand: the three above-mentioned works aim to plainly explain the content that they approach.[9] Furthermore, the title of the first writing indicates from the onset that it addresses novices, while *On Medical Experience*, 2, indicates young people as its preeminent audience, choosing to expose its content in dialog form for precisely this engaging purpose.[10] Therefore, one may appreciate that Galen's style is directed at the type of audience who would find little use for lofty displays of rhetoric.[11]

In fact, the disputes between different medical schools referred to the nature of medical knowledge, to the specific theory of scientific knowledge that can be assumed by the practitioner of Medicine. As early as the Hippocratic Corpus, practice and theory were regarded as fundamental disciplines within Medicine: it was considered that theoretic knowledge about human anatomy and physiology was possible (see the Hippocratic treatise *On the Nature of Man*), and

[9] One should remark how Galen inserts fictional dialogs between representatives of the different medical sects: he makes special use of this stylistic device in the final part of the treatise *On Sects* (8-9), as well as in *On Medical Experience*, passim. He uses a similar rhetorical technique in *An Outline of Empiricism*: toward the end of the first chapter, after discussing the name assumed by the Empiricists, as well as their predecessors, Galen impersonates an Empiricist, in order to render an outline of this epistemological current of thought. "But let us suppose that the person who says all the things which are to be found in this book is an Empiricist" (Galen, *Three treatises*).

[10] Galen provides a most interesting image in the third chapter of this writing: "And now let the Dogmatist speak first, as if he were before the judge in a court of law, ridiculing the arguments of the Empiricist" (trans. R. Walzer).

[11] One should remark the abrupt, highly personal *exordium* of the treatise *On Medical Experience* (1): after he shows himself an adept of the median solution (one that assumes both reason, and experience as instruments in the medical craft), Galen condemns the attitude of Asclepiades, a famous Rationalist. One should remark the exigency of perfect coherence, which, Galen tells us, is not respected by Asclepiades. "He does not make statements which contradict each other only slightly, but employs such as are in startling opposition to one another. If you wish to understand what I mean, consider what you would think of anyone who speaks of experience as something unreliable without the logos, and who asserts that experience does not exist at all, since there is nothing which can appear twice or thrice in the same way, to say nothing of its appearing very many times, as the Empiricists assert. Do you consider these to be contradictory statements or not? I, myself, regard these two views as being absolutely in opposition to one another. Now we find that Asclepiades frequently tries to affirm and maintain each of these opinions, and that he shows much determination in his effort to support and strengthen each one with the help of the other" (trans. R. Walzer).

that doctors should have been able to use these theories for treating the diverse cases they encountered. Famous physicians of Antiquity, such as Diocles, Praxagoras, or Erasistratus proved rather prudent in affirming the primacy of theory, of reason, with respect to medical knowledge: Medicine is a practical discipline, and so theory should be regarded as a corollary aid applied to empirical knowledge. In the 2nd and 3rd centuries AD, however, famous physicians began disputing the nature of specialized knowledge more fervently, several 'sects' appeared, following the theoretical outlines drawn by prominent figures (such as Erasistratus of Herophilus), and the tension inherent to the 'Rationalistic' disposition gave birth to another conceptual option, Empiricism. To put it plainly, the physicians who shared the Empiricist presuppositions considered that the only thing necessary for doctors to conduct their craft was practical experience. As such, the supporters of this view found theory useless: one does not require 'theoretical' concepts (such as atoms, void, pneuma, pores – all empirically imperceptible) to render a correct diagnosis. In order to differentiate their intellectual adversaries, the Empiricists called the latter 'Rationalists.' The natural question that arises is, of course: did the Empiricists allow for the use of reason at all? One should, at least, admit that reason is necessary, if only to capitalize on empirical experience. One requires memory, as well as conceptual correlations, which solicit more than mere experience. It seems that, in the specific technical manner with which they used words,[12] the Empiricists did, in fact, allow for the 'empirical' use of reason (in deductions or inductions), but they by no means accepted theories which affirmed the existence of 'imperceptibles' (hidden causes, atoms). In *An Outline of Empiricism* (XII), Galen relates that the Empiricists rejected the use of reason to transition from theoretic judgment to individual cases, and only accepted to make such a transition to similar sorts of judgment, when it proved necessary.[13] Certainly, the Empiricists claimed, it is allowed to use reasoning, but only usual, 'mundane' reasoning, which arises from empirical experience.[14] This particular type of reasoning had a special name – *epilogism*. It

[12] In *An Outline of Empiricism*, 3, Galen draws attention to how the Empiricists used particular Greek terms with unusual meanings. "These words \i.e. experience, observation\, then, the Empiricist have not used in accordance with Greek usage." In a different fragment, but in the same chapter, Galen notices that the Empiricists were also lexically creative: "In the ancient Greek authors, I have found the word 'somebody-who-has-seen-for-himself' (*autoptes*), but I have not found the word 'one' s-own-perception' (*autopsia*)" (Galen, *Three treatises*).

[13] See, for example, chapter 4 from *An Outline of Empiricism*. Chapter 9 of the same treatise attempts to define 'similarity,' an unequivocal term.

[14] Naturally, the Empiricists' arguments use analogy: they assimilate Medicine to practical crafts (such as ship-sailing, cultivating the land, etc.) or mundane practices, which do not require

can never lead to general, theoretic truths, but only to the resolution of a given situation. The reasoning specific to Rationalists was called *analogism*[15] by the Empiricists.

The disputes between Empiricists and Rationalists must have been epic. One should remark how Galen underlines that the treatment proposed by the disciples of the two schools is similar, and only the ways by which they arrived at this treatment differed.[16] Therefore, the terminus seems identical: the only thing that differentiates the 'Empiricists' from the 'Rationalists' is their underlying theory of science, that which leads them to discover the cure. The information that Galen delivers provide us with a clearer image of the Empiricist perspective. This is because, on the one hand, the Empiricists seem to have reconverted the very Greek terms used for the intellectual debate, and Galen highlights this aspect multiple times. On the other hand, they appear to have structured downright 'battle strategies' against the Rationalists. The first of these – and the most

any special training (e.g. drinking wine, or picking mushrooms.) Chapter 9 of *On Medical Experience* provides multiple examples of this type. On the other hand, considering the internal differences from within the medical sects, a Rationalist could very well argue that the Empiricists are in great difficulty when they claim that medical practice is possible through the experience of observing something "a great number of times." The very expression "a great number of times" is, after all, ambiguous. Moreover, the Rationalists invoked the sorites argument (see *On Medical Experience*, 7): mere accretion of facts cannot generate authentic knowledge. Nevertheless, the Empiricists (especially those who considered that even a single significant experience could lead to a discovery) replied that, if one strictly follows reason, there should be no multitudes in the world, no plurality, so that terms such as "large," "herd," "community" would lack meaning. Furthermore, the *logos* collides with the facts, as such as we arrive to know them through our senses (see *On Medical Experience*, 16-17). As a logical consequence to this *aporia*, they questioned the very possibility of movement within the Rationalists' paradigm (see *On Medical Experience*, 19-22). There is an interesting anecdote about a dispute that Diogenes held with a Rationalist on the topic of movement: after the latter argues that movement does not exist, he replied: "I am surprised at these miserable seafarers who annoy us all day long with their cries of: Who is going to Rhodes, who to Cnidos, to Kos, to Lesbos?" (*On Medical Experience*, 22).

[15] In general terms, "what is known as *epilogismos* is the conclusion pointing to the visible things, and what is called *analogismos* is the conclusion pointing to invisible things" (*On Medical Experience*, 24). In *On Medical Experience*, 25, one may find several more medical examples of arguments with consonant conclusions, but which are evaluated differently by the Rationalists and the Empiricists. As the latter claim, the advantage is that an epilogism is accessible to anyone, whereas not all are capable to comprehend analogisms.

[16] In *On Sects*, 4, Galen explicitly states: "Generally speaking, the dogmatics and the Empiricists use the same treatments for similar diseases. They only dispute the manner in which these remedies are discovered."

important one – was to contest their opponents' theories by using philosophical instruments (mainly taken over from the Skeptics). The second strategy suggested arguing that experience sufficed for the practice of Medicine.[17] Nevertheless, Rationalists, too, had their lines of argument: they invoked against Skepticism some of the greatest names in Philosophy, such as Plato, Aristotle, or Chryssipus, who became staunch intellectual points of reference for the Rationalists. In Galen's *On Sects – for Beginners*,[18] one may find many of the Rationalists' arguments, which he took over from Asclepiades, a famous physician and "Rationalist" in his own right (one who did not share the classical theory of humors, but rather a functional atomism).[19] Following these aspects, what added to the boastful prestige of every sect was to assume quite obvious scientific merits (such as the invention of particular medical instruments or pharmaceutical cures[20]), as well as to claim intellectual descent from ancient authoritative medical figures, in particular, Hippocrates.[21]

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[17] Besides this objection, the Empiricists also reproached the Rationalists that they could not agree on a "single" theory, which was to be expected in the case of those who legitimized their intellectual endeavor as stemming from *logos*: see this stream of criticism in *On Medical Experience*, 11. Hence, the Empiricists concluded that there is no single *logos*, but rather only *logoi*, which makes experience the "unifier" in the sphere of medical knowledge. Of course, Rationalists would try to reply: Galen mentions their response (naïve, in his opinion, as well as that of Alexandros or of one of his disciples), according to which it is possible to discover the same thing through multiple *logoi* (see *On Medical Experience*, 13).

[18] In chapter 5 Galen explains that the "Rationalists" formulated numerous, as well as very diverse replies: some of them contested the fact that experience, as understood by the Empiricists, was possible, others claimed that experience is incomplete, whereas other "Rationalists," still, argued that what the Empiricists proposed is not an "art." For these arguments, please see *On Medical Experience*, 8, where Rationalists' arguments against Empiricists are reduced to three types: some of them denied the possibility of Medicine by the sheer accretion of experience, without the *logos* that, alone, discovers them all, others noted the universal applicability of the induction type professed by the Empiricists, whereas other Rationalists, still, argued that discovery is not possible exclusively by inductive accretion, because the *logos* is also necessary.

[19] A fine Empiricist reply to Asclepiades' objections can be found in the final chapters of *On Medical Experience* (25-27).

[20] For these aspects, see *On Medical Experience*, 26: Rationalists took pride in discovering the catheter, whereas the Empiricists, who considered anatomy useless, excelled in the discovery of pharmaceutical substances.

[21] It was a classical *locus* to assume that Hippocrates was a Rationalist, as "father of Medicine." Nevertheless, the Empiricists also assimilated Hippocrates to their stream of thought, capitalizing, for instance, on *Epidemics*, a work from the Hippocratic Corpus, which comprises several medical "cases" (see *On Medical Experience*, 10). Moreover, the Rationalist

Galen offers little information on the third medical 'sect,' the methodist one. The reason must lie in the outward contempt that the famous physician carried toward this group. Nevertheless, methodism appeared later than the other sects (according to the potential filiation Asclepiades – Themison – Thessalus; the final one is considered the true founder of this medical group). Only in the 1st century AD did methodism fully bloom as a school of medical thought worthy enough to compete with the classical Rationalists and Empiricists. The Methodist doctrine may be described as 'mixed:' it adopts both 'Empiricist,' and 'Rationalist' positions, admitting that theory is a constitutive part of 'medical science.' The Methodists' fundamental theory, that of "obvious communities," described disease as a dilation or contraction, leaving "the method" as the only solution for regaining health (that which is either dilated or contracted, must return to its initial state). One should remark that the Methodists used classical Skeptic *loci* against the 'Rationalists,' following the Empiricists in this respect: the Methodists admit reasoning, like the 'Rationalists,' but they, nevertheless, block it at the level of perception. Galen could not have been a great sympathizer of "the method" (although he carried great respect for particular Methodists, such as Soranus), because the Methodists claimed that the entire medical art could be learned in 6 months.[22] Of course, what appeared outrageous to Galen must have seemed rather chic for the first few centuries AD: an art so complex, such as Medicine, could be learned by anyone in only 6 months. It almost sounds like a modern commercial advertisement.

4. Discussion

4.1 Preamble: Classical Antiquity

Of course, the entire discussion on empiricism and rationalism could not have remained particular to the medical field alone. Indeed, it is within medical writings that one first finds the terms 'Empiricist' and 'Rationalist,' but epistemological debates on the sources of knowledge and the criteria for ascertaining truth exist in other 'arts,' as well. Knowledge in the political sphere, for instance, was considered by the rhetorician Philodemus to be strictly empirical (B, I, 27-28).

thesis, which states the logos' supremacy, makes any attempt to capitalize on medical precedents useless: all can now be discovered by using the logos.

[22] In fact, as Galen affirms in *On Sects*, 6, the Methodists not only part significantly from the classical medical sects, but also defy almost everything in medical practice at the time. In the same chapter, Galen indicates a "blasphemy" of the Methodists: the Hippocratic adage "life is short, art is long" is overturned by the Methodists' interpretation, who affirm that, in fact, life is long, whereas (medical) art is short.

Even in the time of Plato (hence, before the two above-mentioned medical sects appeared), scholars distinguished between physicians who started their practice from general theories on the constitution of the human body, and doctors who started from their own observations on human physiology (*Laws* 720 A-C, 857 C-D). Another art seems to receive the same status in a different Platonic dialog, *Gorgias*. In that text, Socrates affirms that, if one were to follow Polus' and Gorgias' views (which he denounces), one should conclude that rhetoric must be regarded as the fruit of experience (*empeiria*) and of practice (*tribe*), not as an art (*techne*). Therefore, one should conclude that, as early as Classical Antiquity, a debate had arisen with respect to the sources and the nature of knowledge. The precariousness of primary sources, however, makes it difficult to clarify this debate in its finer nuances. Nevertheless, Galen does prove that the debate was perceived as a generic one, beyond the strict scope of one particular *techne* or another. Thus, towards the end of the *An Outline of Empiricism*, the physician discusses the Empiricist intellectual attitude as a generic one: "...since they believe that evident perception and memory suffice for the constitution of all arts."[23]

4.2 Late Antiquity

In Late Antiquity, the epistemological issue becomes acute, and the medical debates of the period testify to this aspect: the differences between the medical 'sects' derived from their argumentative options with respect to the realm of knowledge, and more specifically, with respect to the origins of accurate, reliable medical knowledge. For this reason, it is, of course, natural that fierce polemics would have arisen, and engulfed other specialized fields, as well. Only in modernity did various intellectual disciplines separate rigorously. Thus, it was far easier for an Ancient thinker to be both physician, and philosopher, than for one to do so in our time. Consequently, the intellectual testimonies provided by the medical disputes are worth a lot: the medical writings of Galen survived to a great extent, and they project the features of a complex intellectual landscape. Unfortunately, other areas of knowledge involved in the epistemological debates of Late Antiquity have left far fewer traces. For this reason, it is most difficult to identify 'mirror spots,' which should reflect the intense debates from within the various disciplines. Important philosophical authors do provide us with some snippets that suggest the intensity of epistemological debates. For instance, in *Dissertationes ab Arriano digestae*, 1, 17, Epictetus dedicates an entire section to this issue, and grants it a very suggestive title: *On How Rational Things Are*

[23] Galen, *Three treatises*.

Necessary. The arguments that he presents are of the sort underlying a Rationalist-Empiricist debate.

Heron, the famous scientist of Late Antiquity, also confirms that the distinction between the two lines of thought must have mattered greatly. At the beginning of his treatise, *Pneumatica*, the mathematician insists on mentioning that "the pneumatic matter was considered worthy to be examined by the ancient philosophers and mechanics." In the following sequence, where he classifies all writings on this subject, Heron states: "some have discussed the potency of this matter from a rational perspective (*logikos*), whereas others also [discussed] it according to the action of perceptible things" (see *Introduction*). Consequently, for Heron, one could expose the study matter either from a strictly Rationalist perspective, or a opposed one. This manner of interpreting the specificity of a scientific discipline seems to have been somewhat common, even banal, since it is mentioned in the preamble to the scientific treatise.

The question of sources for scientific knowledge was also discussed (by reflex) in another prominent work from Late Antiquity. Porphyry, the 3rd – century Neoplatonist philosopher, focused not only on what any History of Philosophy treatise indicates to us today, but also on other fields, such as Philology and Music. In a commentary that he makes on the *Harmonics* of Ptolemy, Porphyry remarks the dual nature of Music, relying, as criteria for his argument, on multiple authoritative figures, or other *loci* invoked in the works of Ptolemy. Thus, he introduces "The Musician Didymos,"[24] an entry in which he discusses what defines truth. Here,[25] Music not only constitutes a rational doctrine (*logikon mathema*), but also one that is simultaneously sensory, and logical (28). Porphyry argues that this conclusion arises because there are also two criteria for truth: both senses, and reason. More importantly, as the treatise follows this same criterion, authentic

[24] It is not at all easy to identify this figure. The name "Didymos" was widespread in Antiquity. If we follow the *Suda Lexicon* (under the heading "Didymos"), we find that there was a certain Didymos, son of Heracleides, who was passionate about Music, and who was a contemporary of Nero. This is the only "Didymos" whom one may find in the *Suda* with the explicit characterization "mousikos". *Pauly-Wissowa* considers him a distinct person (see Didymos, 11), whereas *Der … Neue Pauly* attributes to Didymos of Alexandria, the famous philologist and polyhistor, the musical work previously attributed to Didymos the musician.

[25] The fragment cited by Porphyry comes from the work *On the Difference between the Disciples of Aristoxenos and those of Pythagoras*. As I mentioned in the previous note, this work has an uncertain author: *Pauly-Wissowa* attributes it to a person mentioned by the *Suda* (see *Didymos*, 11), whereas *Der... Neue Pauly* attributes it to Didymos of Alexandria, commentator, scholiast, polyhistor (who conducted his scholarly activity at the beginning of the Christian Era): the latter is mentioned in *Pauly-Wissowa* as a distinct person, with a sizable work (see *Didymos*, 8).

knowledge, the treatise gradually introduces its reader into the history of the discipline. Thus, one first learns (26) that some of those preoccupied with Music tried to exclude reason from their research as much as possible, and took into consideration only what their senses provided. This tendency outlines itself in their systematic refusal to argue by using reason or logical consequence, considering that the mere sensory preoccupation (*tribe*), which is born out of custom, would suffice. Didymos exhibits his derogatory opinion on such musicians: he deems them rudimentary, and he underlines that they are merely interpreters of Music, not musicians proper.

Porphyry's text also provides an important piece of information: this type of musicians still existed in the time of Didymos, and they were the ones who engaged in "preoccupation without reason" (*alogw tribe*). Moreover, as Didymos claims, there were other musicians, who preferred reason as their sole criterion, nevertheless using sensory information as a starting point in their discourse. They are identified with the Pythagoreans, who admit to perceptible, sensory elements, only to the extent that they do not contradict reason. Finally, Didymos also discusses a third category of thinkers who theorized on music, namely those who granted equal importance to reason and sensory experience. Amongst them, he especially distinguishes Archestratos. Of course, it was possible to take a nuanced intellectual stance within a wide spectrum, and Didymos mentions Aristoxenos as an example in this respect. The latter suggested that both types of criteria were necessary within their respective, separate fields (sensory/intelligible). Nevertheless, he preferred empirical knowledge in certain situations, which, as he argued, could by no means be supplanted by theoretical constructions (27-28). Once again, it is important to underline the essential role that the discussion on the criteria for finding truth plays in this discussion. Didymos recounts that this discussions was part of the first book of Aristoxenos' treatise, *Harmonic Elements.* At the end of the long quote from Didymos, Porphyry feels the need to explain: although the passage suggests that Ptolemy and Aristoxenos convened on their opinions, that was not so. In fact, Ptolemy took the notion of "reason" from the Pythagoreans, whereas the concept of "sensibility" comes from Aristoxenos. Porphyry considered this theoretical position as "mixed" (*meiktos*), or "elective" (*kat'eklogon*).

It comes across that the epistemological problem was important from another fragment cited by Porphyry: the source was Ptolemais of Kyrene, who, in all semblances, synthesized the entire musical doctrine of Classical Antiquity. It is also very important to notice the place where the author discusses the musical 'sects:' she refers to the differences between them right from the onset, in the work's introduction.

Another important testimony about the Rationalist-Empiricist pairing comes from Sextus Empiricus, where yet again, it is oriented toward a different scientific field. In the first book of his *Aduersus Mathematicos*, which was dedicated to the study of grammar, the author defines this discipline of language in accordance with Dionysus the Thracian: „grammar is, for the most part, the experimental knowledge (*empeiria*) of that which was said by poets and writers" (1, 57). After he exemplifies what the study of "grammar" entails, by invoking names of relevant authors, Sextus Empiricus cites an objection brought against Dionysus: "Ptolemy the Peripatetician, however, challenges him, by affirming that he should not call grammar experimental knowledge (*empeiria*) – for experimental knowledge is a particular occupation (*tribe*), and it is a work wholly lacking in art and reason, which consists of mere observation and practice, whereas grammar is an art" (1, 60). Of course, it is difficult to identify the famous philologist's addressee: *Pauly-Wissova* avoids placing the Peripatetician within a particular time frame (see *Ptolemaios*, 70). In any case, the debate must have actually taken place, since the author who provides us with this information, Sextus Empiricus, goes on to explain that he replied to Ptolemy's allegations in his work *Empiricist Memoires*, and that he formulated his response in the manner of Metrodorus: *empeiria* is, actually, a synonym for *techne* (1, 61). Once more, one may notice how the debate on the origins of knowledge is framed between 'Empiricists' and 'Rationalists,' of whom the latter considered grammar to be a *techne*.

5. Conclusions

All that I have discussed above attempts to reconstruct an intellectual landscape in which the epistemological debate was essential. Either implicitly, or explicitly, the problem of the very possibility of knowledge, of its sources, or of the criteria for truth, stirred the acute interest of thinkers in Late Antiquity. Heated debates raged throughout the period, questioning precisely the Epistemology that one assumed. As shown, the specific arguments involved reach beyond the domain of individual 'arts,' and indicate generic intellectual options within the field of knowledge. Several prominent figures of the philosophical sects are recognized as philosophers in the common-speak of the discipline: Menodotus and Sextus, dubbed "Empiricus" ("the Empiricist") support the views of Pyrrhonian Skepticism.

ARISTOTLE *VERSUS* VAN TIL AND LUKASIEWICZ ON CONTRADICTION: ARE CONTRADICTIONS IRRATIONAL IN SCIENCE AND THEOLOGY?

Robert C. TRUNDLE

ABSTRACT: The Polish logician Jan Lukasiewicz and the American theologian Cornelius Van Til are famous for challenging Aristotle's Principle of Contradiction. Whereas apparent contradictions such as God and physical reality being both One and Not One (Many) are accepted in terms of an *idealism* held by Van Til, the Principle's violations in theology and science reflect a *realism* held by Lukasiewicz. Lukasiewicz is favored for explaining why the Principle's violation may be rational for a scientific and theological realism.

KEYWORDS: Aristotle, contradiction, Jan Lukasiewicz, Cornelius Van Til, science, theology

Two central 20th-century challenges to the Principle of Contradiction are those of the Polish logician Jan Lukasiewicz, that bear fruitfully on physics,[1] and those on metaphysics and the Trinity by the American theologian Cornelius Van Til. Whereas Van Til defended an orthodox Calvinism against a doctrinal abandonment of such things as the virgin birth of Christ by the Princeton Seminary where he taught in the 1920s,[2] Lukasiewicz was a devout Roman-Catholic logician who in 1910 defended a view of the limits of logic that are reminiscent of St. Augustine (for whom immutable rules of inference should be

[1] Some violations of the Principle of Contradiction are consistent with scientific realism, revealing ignored faults with a realism of Sir Karl Popper, despite Popper's possible support at some level by Lukasiewicz, *e.g.* a "conjectural conception very close to Popper's…" See Fran Coniglione's "Filosofia e scienza in Jan Lukasiewicz," *Epistemologia* 17:1 (1994) 73-100.

[2] See John Frame's "Cornelius Van Til," *IVP Dictionary of Apologetics* at http://maritain.nd.edu/jmc/ti99/pouivet.htm, 7 Nov 2007.

distinguished from truth about reality[3]); in spite of Lukasiewicz's relation to the Lvov-Warsaw School that stressed modern logic in Aristotelian argumentation against a rising anti-realist and anti-metaphysical irrationalism – political and positivistic.[4] Intriguingly, his suggestion that logical reasons do not provide the strongest motive for belief brings to mind Ludwig Wittgenstein, notwithstanding his early affiliation with the Vienna Circle, who suggested that we too often follow a rule *blindly*.[5]

Consider Lukasiewicz's doubts about blindly adhering to the Contradiction Principle after summarizing misgivings about the Principle by Van Til regarding the Trinity. This paper shall then seek to show, among other things, that the Trinity's possible violation of the Principle proceeds *pari passu* with its reasonable contravention for solutions to knotty epistemological problems in the philosophy of science.

I. Van Til's Misgivings about its Application

After his youthful affiliation with the Christian Reformed Church, Van Til (1895-1987) moved from Holland to America where he attended Calvin College and, later, the Calvin Theological Seminary. He transferred to the Princeton Theological Seminary for his Th.M. in 1925 and gained his Ph.D. in 1927 at Princeton University where his dissertation compared Reformed Theology's notion of God with the "*absolute* of philosophical idealism."[6] When one renowned

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[3] St. Augustine, *On Christian Doctrine*, trans. D.W. Robertson, Jr. (New York: Macmillan Publishing Co., 1988), 72.

[4] I am indebted to Roger Pouivet for this insight. But some of his views *may* differ from mine on both Lukasiewicz, who I may construe more liberally (epistemologically), and W.V. Quine who is noted briefly below. See Pouivet's insightful "Faith, Reason, and Logic," *Jacques Maritain Center: Thomistic Institute* at http://maritain.nd.edu/jmc/ti99/pouivet.htm, 7 Nov 2007. He notes that the "Lvov-Warsaw School was the major influence within Polish philosophy between the two world wars… Among the distinguished philosophers of this school are Tadeusz Kotarbinski, who was closely studied by Peter Geach, and Alfred Tarski… [Also] Quine's sojourn in Warsaw during this period had a very strong influence on his thought and… through him the ideas of the Lvov-Warsaw School were subtly osmosed into a large part of so-called analytic philosophy."

[5] Pouivet, "Faith, Reason, and Logic," online without page/section numbers.

[6] Cornelius Van Til's *An Introduction to Systematic Theology* (Syllabus, 1961), 11, and *A Christian Theory of Knowledge* (NJ: Presbyterian & Reformed, 1969), 202, emphasis. From John Frame's "Van Til: The Theologian," Center for Reformed Theology & Apologetics, http://www. reformed.org, 20 Nov 2007, n. 108. The following accounts are from Van Til's works, referenced by Frame who notes that Van Til's contributions are "of virtually Copernican dimensions."

scholar notes that his "contribution to theology is of virtually Copernican dimensions," we might infer that the dimensions are inspired by, if not loosely analogous to, the idealism of Kant's Copernican Revolution in Philosophy: As this Philosophy held that our mind imposes categorial interpretations on reality wherein reality is not known in itself, our mind's limited ability to know reality reflects Van Til's idealism. And as the idealism advanced by Kant led to *Weltanschauung* Analyses with 'truth' being incoherently relative to rival worldviews (less Kant's categories in the philosophy of science such as those of Thomas Kuhn and Paul Feyerabend),[7] we may anticipate comparable problems for theology and metaphysics – as well as for physics by implication – in Van Til's idealism. This idealism, admits John M. Frame who favors it, begets difficulties in assessing Van Til's understanding of logic.[8]

Though it is not always clear when 'logic' means formal logic, versus an idealist method of thought, "God is not subject to some source of (logical or other) possibility more ultimate than himself."[9] Rather, God himself determines ultimate possibilities and both "vindicates and limits the competence of human logic" so that His revelation contains "no 'real' contradiction."[10] Though contradictions in Scripture may be apparent, believers should know that there are no contradictions from God's viewpoint. And this viewpoint cannot itself be inconsistent, says Van Til, because God is the very foundation of logic whereby "Logic itself does not determine what is possible or probable; only God does that."[11] God's determining the probable and possible results in distinguishing theistic from non-theistic secular thought.

A. Non-Theistic and Theistic Thought

Whereas the *non-theist's self-centered mind* has a univocal or one-dimensional grasp of "analysis and synthesis, correspondence and coherence, objectivity and

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[7] *Weltanschaunng* (Worldview) Analyses superseded Kant's *a priori* categorial interpretations common to the human race by those of different races, genders, cultures etc. Cf. Frederick Suppe, ed., *The Structure of Scientific Theories* (Chicago: University of Illinois Press, 1977), 126, n. 258.

[8] Van Til's *A Christian Theory of Knowledge*, 202 and *An Introduction to Systematic Theology*, 11. From Frame's "Van Til: The Theologian," fn. 108.

[9] Van Til's *A Christian Theory of Knowledge*, 202 and *An Introduction to Systematic Theology*, 11. From Frame's "Van Til: The Theologian," fn. 108.

[10] Van Til's *The Defense of the Faith* (Philadelphia: Presbyterian & Reformed, 1955), 61f. and *A Christian Theory of Knowledge*, 38. From Frame's "Van Til: The Theologian," fn.111.

[11] Van Til's *A Christian Theory of Knowledge*, 37 and *The Defense of the Faith*, 228. From Frame's "Van Til: The Theologian," fn. 117.

subjectivity, a priori and a posteriori, implication and… transcendental versus syllogistic reasoning," says Van Til, the *theist's God-centered mind* grasps "any or all of them analogically."[12] While we are related analogically to God since we are like Him *inter alia* by being limitedly rational and unlike Him because He is rational in a supereminent (infinite) way, according to St. Thomas, the analogical reasoning held by Van Til means in part that we are like God in aspiring to reason logically but unlike Him in illogically thinking that He must conform to our norms of rationality. Rational norms are His creations and a created Principle of Contradiction can be violated *prima facie* in our minds but not necessarily in the mind of God. God determines what is in fact contradictory. While contradictory claims of science can be regarded *literally* as incoherent and be denied, apparent contradictions in the sacred domain of God's revelations are construable only *figuratively* as being incoherent; the incoherence better depicted, perhaps, as a *mystery* that passes human understanding – an understanding proper to God that exceeds our noetic limits and is, consequently, outside all of our cognitive boundaries.

These boundaries resulted in Van Til comparing the Greek quandary of Reality as One and Not-One (Many) to God being One and Not-One (a Trinity). The Trinity is no more contradictory in the theist's mind than the metaphysical anomaly posed by Heraclitus and Parmenides. Whereas Parmenides held that the Many observable things must be illusory because their change implies the impossibility of *being* coming to *be* when it already *is* and going out of being when there is nowhere to go, Heraclitus argued that an unchanging One is illusory because its inference from a *thingness* common to the many *things* results in the Many being both changing and unchanging. Surely a truth-claim that the Many are real may strictly be more certain epistemologically than an inferred reality of the One, as by analogy some logicians might say that the truth of the conjunction $p \wedge q$ is stronger epistemologically than that of the inferred proposition p in terms of the inference rule of simplification $p \wedge q \mathbin{/\!/} p$ even though p is entailed logically. At the same time a dilemma arose inasmuch as the One, besides being inferable from the Many, rendered coherent the approximate truths about many changing things in virtue of these things limitedly being forms or manifestations of the unchanging One.[13]

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[12] Cornelius Van Til, *A Survey of Christian Epistemology*, Vol. II of *In Defense of Biblical Christianity*, 2nd ed. of the Syllabus (Philadelphia: Presbyterian & Reformed Publishing Co., 1932), Ch. 1. See http://www.reformed.org/master/index. html, 20 Nov 2007.

[13] Thales held problematically that the changing Many were *literally* forms of an unchanging One, his one-and-many paradigm influencing Plato. Yet Plato held that the Many in the

That is, without a reality of the One as a principle of identity for the many observable things in change, any changing thing would already *not be* what it is claimed to *be* and there could be no inexact truths about anything that there in fact are. Thus although reality being One and Many is ostensibly contradictory, the alleged contradiction is analogous to the Trinity. By relating the "Trinity to the problem of the one and many," states Ralph Smith, Van Til places "the Trinity at the foundation… of all thought about any and every subject."[14] Though the realities of the One and Many are contradictory in terms of the non-theistic univocal mind, the contradiction is only apparent for the theistic analogical mind.[15] And although this mind would not evidently accept that all seeming contradictions are divinely underwritten because some can be resolved theologically, philosophically and scientifically, the theistic mind appreciates by the grace of God that various creations of mystery are analogues of a mysterious Creator.

But the Creator's creations only *seeming* to be contradictory may be problematic for Van Til's idealism. Idealism is a philosophy, so a question arises: Should philosophy inform religion or religion philosophy? On the one hand, idealism is disregarded by most orthodox Christian views that accept God's revelation of Himself in the New Testament, without any philosophical caveats, as a Trinity. Genesis in the Old Testament presages this Trinitarian anomaly when God said, despite a Hebraic monotheism, "Let *us* make man in our image…" (1:26, emphasis). In virtue of this holy image, on the other hand, Van Til would presumably reject incompatible attributes being *properly* ascribed to either our selves or other creations (especially an ultimate analogous creation of the One and Many); a revealed goodness of the creations being related furtively in Genesis 1:31 to their not being unintelligible, and ideally to their being rational, and therefore also to their not violating the Contradiction Principle.

Visible World *shared only limitedly* in a hierarchy of increasingly universal unchanging Forms, culminating in an ultimate Form (One), in an Invisible World. These two Worlds were criticized by Aristotle since unchanging Forms, say the Form Man, cannot be related coherently to changing particulars, say the particular man Socrates: the words 'sharing in' being poetical. And so without abandoning 'forms' as objects of knowledge, Aristotle argued for only one world of particulars that are unities of matter and form wherein the forms man or woman are fully in particular persons.

[14] R.A. Smith, "Van Til's Insights on the Trinity," http://www.trinitarianism.com (26Nov2007) 7.

[15] The 'one and many' contradiction is not deemed irrational by Karin Verelst (Math Department at Vrije Universiteit Brussel) and Robert Coecke (EPSRC Research Fellow in Quantum Computer Science) in "Early Greek Thought and Perspectives for the Interpretation of Quantum Mechanics" (*arXiv:physics/0611064* v1 2006): "abandoning the principle of contradiction implies the loss of *neither* the capacity to reason soundly *nor* the possibility to use mathematics [12, emphasis]."

How can the Principle be inviolable logically and sacrosanct theologically, however, and still be reconciled with other traditional insights? What of St. Augustine's insight that we are immediately and incontrovertibly aware of our existence as both *one* and *not one* inseparable life ("being, knowledge, and will" that are logically distinct)?[16] The one and many of our own existence are, he says, "far from [a holiness of] the Trinity, but I suggest them as a subject for mental exercise…"[17]

B. A Realism that is Unlimited by Idealism

The exercise posed by Augustine, in terms of which the Trinity is analogous to our triune existence, lends itself phenomenologically and logically to a realism that is unlimited by idealism:[18] Since there can be no thought without a consciousness of it but there can be consciousness without thought – thought not being as fundamental as consciousness, reality need not conform to reason. There is no reason for the theistic scientist to dismiss violations of the Principle of Contradiction when contradictory notions are either revealed in Scripture or inferred from an experienced reality. And so in the spirit of physicist and theologian John Polkinghorne, the theist might say, "all forms of realism are divinely underwritten, for God will not mislead us..."[19]

Being misled from a theistic standpoint, in terms of Augustine's realism, may suggest prophetically that to reject the Trinity since it violates the Principle of Contradiction is to reject the reality of our own triune existence. And if this existence is understood as our *mind*, *body* and *free will*, the theist might infer that philosophers who impose the Principle on God will impose it also on both an external reality and themselves; resulting, for instance, in an idealism of Hegel

[16] St. Augustine, *Confessions*, trans. R. Pine-Coffin (Middlesex, England: Penguin Books Ltd, 1984), 318.

[17] Augustine, *Confessions*, 318.

[18] Phenomenology, following Husserl, Heidegger, Sartre and others has focused *inter alia* on our consciousness of reality to distinguish reality from our thought. Intriguingly, St. Augustine pioneered phenomenological explanations of how our thought and free will are related to consciousness: To be conscious of our behavior is to be implicitly conscious of our will to behave or not to behave in given ways; as to be conscious of our thought is to be implicitly conscious of our freedom to think or not think, including the self-refuting thought that all thoughts are caused (our knowing they are not caused being rooted in our immediate consciousness)! See, for example, my "St. Augustine's Epistemology: Ignored Aristotelian Themes and Their Intriguing Implications," *Laval Théologique et Philosophique* 50:1 (1994): 187-205.

[19] John C. Polkinghorne, *The Faith of a Physicist: Reflections of a Bottom-up Thinker* (MN: Augsburg Fortress Press, 1994), 156.

that stresses only the *mind*, a thoroughgoing materialism of Marx that admits only of physical *bodies*, and an unfettered *free will* and will-to-power-to-truth held by Nietzsche that presaged an existentialist dictum that 'truth' is posterior to our existence – that 'existence precedes essence' wherein truth about reality is a function of, or determined by, willful *Übermenschen* (Supermen).

These philosophical winds of doctrine, theists can add, are warned against in Scripture and beg for other distinctions of realism from idealism. The idealism advanced by Van Til regards logically necessary truth as a function of God's mind; what He determines. But insofar as the determination applies to less esoteric experience such as metaphysical issues of the One and Many, he prescribes a *limited rationalistic realism* for our minds. That is, while the mind of God excludes literal contradictions for both His revelations and select analogical anomalies in metaphysics, Van Til acknowledges more mundane contradictory claims about a secular reality that are not just apparent and that should be denied by the theistic mind. In other words, this mind can perceive a physical reality that both *is* as it *is* apart from our thought and cannot have logically incompatible attributes. Indeed, unless Van Til accepted such a *limited rationalistic realism*, he would not have been so gravely concerned to avoid contradictory revelations about reality.

Realists who are theists, without the idealism and rationalism, by contrast, can reason unreservedly from an experienced reality and revelations to their ideas. This holds even if the ideas ascribe incompatible properties to something, illustrated by Augustine when he infers inconsistent attributes of our existence from our conscious experience: To accept experiential contradictions is to accept revelations such as the Trinity and vice versa since the revelations and reality are conditions for the truth of our truth-claims. This epistemic approach is not only rational but also fruitful for explaining experienced incongruities that perennially typify the human condition. In addition to an experience of our existence being both one and many, for instance, our immediate awareness of ourselves reveals that we are both free and not free: *not free* insofar as we have bodies subject to deterministic laws of science and *free* insofar as to be conscious of our behavior is to be implicitly conscious of our freedom (free will) to behave in given ways.

For example, we are aware both of being caused to fall on a bus if it stops suddenly and of our will to not fall by grabbing a rail.[20] And this everyday example explains judicial systems where prosecutors blame defendants for freely chosen criminal behavior but the same behavior is held viably by the defense to be mitigated by scientifically understood 'root causes,' causes weighed by judge or

[20] Cf. my "Paradoxes of Human Nature," *Ethics & Politics/Etica & Politica* 9, 1 (2007): 181-186, http://www.units.it/~etica.

jury against freely-chosen behavior as evidenced by forethought. Also, these points are related to retributive and rehabilitative theories of punishment as well as to ideologies of conservatives and liberals. Liberals tend to stress psychobiological causes of our behavior; our behavior being largely grasped by conservatives in terms of a responsibility that presupposes free will. Thus a benefit of recognizing our being limitedly both free and not free, regarding typical behavior, is an implied prescription for moderation in today's political polarizations.

Must these practical considerations, from politics to human nature, be euphemistically treated as mysteries or unproblematic paradoxes?[21] Is it self-contradictory to assert that reality *need not* abide by the Contradiction Principle? The illogical is to impose the Principle on reality, to insist that reality reflect our reason. Thus it may be truer to say that theistic *idealists*, as much as atheistic univocal thinkers, may have that irrational thought due to a self-centered mind. And with this dramatic reversal there could be traditional concepts of sin in accord with the *realist* who may have a God-centered mind: The sin of pride need not be committed of expecting revelations, the reality created by God, and God Himself to conform entirely to human reason.

In the foregoing senses the unadulterated realism seems preferable, metaphysically and epistemologically. The realism is simpler, permits more straightforward inferences from both physical reality and Scripture, and is more cogent than Van Til's irregular mixture of realism and idealism in order to reject contradictory realities in some cases but accept them in other cases as merely apparent.

Indeed, would not seeming contradictions be assessed differently by different theistic interpretations in a way akin to interpretative *Weltanschauung* Analyses in the philosophy of science? In parodying the competing scientific worldviews there could be norms for "objectivity and subjectivity, a priori and a posteriori, implication and linear inference,"[22] to use Van Til's words, that are grasped analogically in inconsistent ways by theistic worldviews of *God-centered*

[21] A mere paradox posed by Jean-Paul Sartre is that "one suffers… from not suffering enough" as when military recruits must suffer a senseless brutality to prevent a more brutal suffering on the battlefield. Or politicians may insist that an increasing buildup of weapons begets a decreasing risk of their use. See Sartre's *Being and Nothingness* (New York: Philosophical Library, 1956), 91. For the weapons paradox, see the U.S. Catholic Church's reference to that seeming illogicality in the *Catechism of the Catholic Church* (Citta del Vaticano: Libreria Editrice Vaticana, 1994), 557, # 2315.

[22] Cornelius Van Til, *A Survey of Christian Epistemology*, Vol. II of *In Defense of Biblical Christianity*, 2nd Ed. of the Syllabus (Phillpsburg, NJ: Presbyterian & Reformed Publishing Co., 1932), Ch. 1. See http://www.reformed.org/ master/index.html, 20 Nov 2007.

minds. Though this *mind* in the best sense unquestionably characterized Van Til, there seems to be no question also that spiraling schisms since the Protestant Reformation undercuts the notion that these minds would agree about either norms or their applications to reality and revelation.

C. Epistemic Objections to the Principle's Violations

One may object that not either the Trinity or our triune existence, among other well-known anomalies, actually violate the Principle of Contradiction. As classically expressed by Aristotle, for example, the Principle specifies that "the same attribute cannot at the same time belong and not belong to the same subject in the same respect."[23] And Aristotle explicates this elsewhere in terms of the impossibility either for "anything at the same time to be and not to be"[24] or for it to "be at the same time true to say the same thing is a man and is not a man [a donkey, in an example used by St. Thomas Aquinas]."[25] But in addition to Søren Kierkegaard (if not traditional theology) stressing that Christ was fully both man and not man (God) – being the insoluble paradox that Kierkegaard intended only if the Principle of Contradiction is rationally inviolable,[26] many other contradictory positions are strikingly reminiscent of St. Thomas.

Regarding God's creative omnipotence, Thomas asserts that everything is "absolutely possible" in regard "to the idea of '*being*' except '*non-being*'."[27] To say that Christ was man and not man (God), or that I am aware of my self as both being and not being one inseparable self (mind, body and will), or that reality is both One and Not-One (Many), or that God is both one and not one Being (a Trinity) would be contradictory insofar as they mean that a given *being* ψ has the attributes φ and non-φ. Yet this contradiction does not mean either that ψ (*being*) is non-ψ (*non-being*) or that the attribute φ both has and does not have *being*, the evident point made by Thomas.

Clearly, Thomas entertained viable violations of the Principle, despite some ambiguity (*Summa* I, 25, 3 and I, 45, 2) when he echoed Christ's answer that

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[23] Aristotle, *Metaphysics*, Bk. IV, Ch. 3, 1005^{b}, 19-20.

[24] Aristotle, *Metaphysics*, Bk. IV, Ch. 3, 1006^{a}, 2-3.

[25] Aristotle, *Metaphysics*, Bk. IV, Ch. 3, 1006^{b}, 32-34, and St. Thomas Aquinas, *Summa Theologica* I, 25, 3.

[26] Cf. Søren Kierkegaard, *Philosophical Fragments*, 62 (SV1 IV, 227). From Jyrki Kivelä's "Kierkegaard on Miracles," *Søren Kierkegaard Newsletter* 43, Feb 2002. Kivelä says, God being paradoxically human is "not *an*, but the object of faith." For the paradox being a contradiction, see L. R. Horn's "Contradiction," *The Stanford Encyclopedia of Philosophy* (2006), ed. Edward N. Zalta, http://plato.stanford.edu/archives/fall2006/entries/contradiction.

[27] St. Thomas Aquinas, *Summa Theologica* I, 25, 3 (emphasis).

"With God all things are possible" (Mark 10: 25-27) in reply to the impossibility of a camel passing through a needle's eye. Christ's answer reflects Thomas' assertion "Not only is it possible that anything should be created by God, but it is necessary to say that all things were created by God…" in reply to the objection that He is not omnipotent since He could not make "the whole to be less than its part" and "affirmation and negation [to be] true at the same time."[28] So to say that one is aware of one's self as both one and not one life (body, mind and will) or that Christ was fully man and not man (God) is not to say that something is being and non-being. Non-being is akin to *nothing* and being to *something*. Yet the above violate the Principle since a subject has logically incompatible predicates: not potentially, but actually at the same time and in the same respect.

D. Is it Heresy to Contravene the Contradiction Principle?

Do the Principle's contraventions amount to heresy in traditional Christianity, influencing logic and the philosophy of science? Many theologians may say that the violations do exactly that because they radically divorce reason from faith: An irrational leap of faith must be made, one may say, if God flouts our most sacred of all principles. But a central point herein is the reasonableness of reasoning *from* an experienced reality, a reality that believers believe was created by God, *to* our ideas and not vainly impose our ideas, even ideas of logic, on reality. And if it is senseless to demand that physical reality abide by the Contradiction Principle, then it is nonsensical *a fortiori* for religious believers to insist that the Principle be obeyed by God. Indeed, if God is ultimate Reality, this realism appears well suited to the compatibleness of reason and faith.

Faith can be reasonable without reason sustaining it, theists can say, in virtue of one's palpable experiences of Love (agape), sin and guilt. The existential point has been made perennially that one *feels guilt* when one behaves wrongly, *not irrational* in terms of a rationalistic ethics that is principally concerned to abide by the Laws of Thought. And theists can note that sin and Love refer to religious, not philosophical, notions that are inexplicable without a loving personal God who alone can forgive sins. The theist can add that although sin is a *mystery*, unfathomable to logic, an alleged logical impossibility of the Trinity need not be euphemized by either the word 'mystery' or Van Til's analogical thought to make God more agreeable to human thought.

Van Til's efforts to mollify rational believers by qualifying his critique of the Principle of Contradiction, by postulating that its violation is merely apparent

[28] Aquinas, *Summa Theologica* I, 45, 2.

in certain revelations and analogous metaphysical problems, appear to support Lukasiewicz's point about an inordinate influence of '*the* Philosopher' Aristotle; despite an Aristotelian-Thomistic tradition that augmented philosophically his devout Roman Catholicism. Having exalted the Principle "as the highest law of thinking and being" in terms of "a stubborn polemic, in which indignation and contempt is vibrating," Aristotle, he said, "persecuted all those who would not accept this law (I skip over their names)."[29]

II. The Logical Misgivings of Lukasiewicz

This harsh criticism of Aristotle and the Contradiction Principle beg for some perspective. Suggesting that Pope John Paul II may have properly ignored the Polish Thomists when other Thomists were praised in his encyclical *Fides et Ratio*,[30] Roger Pouivet credits Lukasiewicz with countering a confusion of the Polish Thomists over faith's relation to reason. In largely reducing reason to a rigor of formalized logic, they muddled a tradition that extended down through St. Thomas: Logic did not account for either acts of belief (*consideratio*) or belief's free acceptance (*assentire*).[31] Assenting freely and free will are, indeed, necessary conditions for religious belief; believing revelation requiring also the supernatural gift of grace (Thomas' *Summa* (II-IIae 5, 2).[32]

Yet we may reasonably suppose that to have the grace to believe revelations is to have the grace to believe not only in a revealed Trinity, even if it violates the Contradiction Principle, but also in the Principle's possible violations in metaphysics and physics. Given the modern revolution in physics with its assumed rationality amidst a mounting cascade of epistemic problems, one might empathize with Lukasiewicz's seeming harshness. And one might suppose that his cynicism would have been more appreciated by philosophers of science, even more in Poland by the Polish Thomists.

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[29] Those who Aristotle persecuted include some living Sophists in Aristotle's time and maybe to Heraclitus posthumously. See Owen LeBlanc's "Lukasiewicz, Aristotle, and Contradiction," *Papers on Lukasiewicz*, http://www.fmag.unict.it/~polphil/Polphil/Lukas/LeBlanc.html, 29 Nov 2007, Section 1.3. The passage is from the 2nd draft's first page of Lukasiewicz's English translation of *0 Zasadzie Sprzecznosci u Arystotelesa: Studium Krytyczne* (Krakow: Polska Akademia Umieijetnosci, 1910; rev. and ed. by J. Wolenski, Warsaw: Panstwowe Wydawnictwo Naukowe, 1987– German trans: Lukasiewicz [1993]); titled *On the Principle of Contradiction in Aristotle: A Critical Study*, the translation starting on 6 April 1955.

[30] The non-Polish Thomists include Jacques Maritain and Etienne Gilson.

[31] Pouivet, "Faith, Reason, and Logic."

[32] Pouivet, "Faith, Reason, and Logic."

It is beyond my scope to delve historically into these Thomists whose rigorous view of reason is outlined concisely by Pouivet. Without accepting his criticism that they fostered a parody of religious belief since they sought logical proofs of God,[33] this God being one of Natural Theology (implicit in the Theology of a Supernatural God via Scripture such as Job where God is a Causal Creator),[34] readers who seek further exegesis can peruse his erudite essay. Suffice it to say that the Thomists included famous logicians such as Jan Salamucha, Boleslaw Sobocinski, Josef Bochenski and Jan Drewnowski who founded the Cracow Circle in 1936 at the 3rd Polish Conference in Cracow. Though the Cracow Circle was influenced by a logical analysis and anti-psychologism of the earlier Lvov-Warsaw School with the membership of Lukasiewicz, he raised the question of why laws of logic must be followed blindly. "Logical analysis," says Pouivet, "doesn't answer this." Such rules "have not been proven logically" but rather "are basic, entrenched instruments of our thought."[35] However, "Lukasiewicz recognized that logical reasons do not provide the strongest motive for believing something…" – something of the kind being indicated by Wittgenstein "when he declared that we follow a rule blindly."[36]

A. Did the Later Wittgenstein Echo the Earlier Lukasiewicz?

While Wittgenstein viewed religious belief "as based on qualities of character… he did not himself possess," he "revered the writings of St. Augustine" and held Kierkegaard in "awe… as a 'really religious' man."[37] These personal qualities bear on belief in religion, science and logic alike since Kierkegaard and Augustine were impatient with logical proofs and claims about irrefutable logical principles. A rigid belief in them is addressed in Wittgenstein's *On Certainty* in a way reminiscent of Lukasiewicz's earlier qualms about logic supporting belief. Thus Wittgenstein says, "I believe that every human being has two human parents; but Catholics believe that Jesus only had a human mother."[38] And they believe that

[33] Pouivet, "Faith, Reason, and Logic."

[34] *Modal logic* replaced a *truth-functional logic* for the proofs. The proofs, for John Paul II, are "the point of departure for… Kant" who rejected the approach "of the Bible and of Saint Thomas Aquinas," per *Crossing the Threshold of Hope*, ed. by V. Messori (New York: Alfred Knopf, 1994), 34.

[35] Pouivet, "Faith, Reason, and Logic."

[36] Pouivet, "Faith, Reason, and Logic."

[37] Norman Malcolm, *Ludwig Wittgenstein: A Memoir*, 2nd ed. with Wittgenstein's Letters to Malcolm (New York: Oxford University Press, 1984), 59, 60.

[38] Ludwig Wittgenstein, *On Certainty*, eds. G.E.M. Anscombe and G.H. von Wright, trans. Denis Paul and G.E.M. Anscombe (New York: Harper Torchbooks, 1969), 32e, #239.

wine becomes the blood of Christ: "if [G.E.] Moore said 'I know that this is wine and not blood,' Catholics would contradict him."[39]

Does the contradictory doctrine of Catholics contravene the Contradiction Principle? Cannot the Eucharist be both wine and not wine, with Christ's presence not excluding the presence of wine,[40] as much as traditional Christians may understand Jesus as being both man and not-man in virtue of his father being the Holy Spirit and his mother human? Is not the disbelief in Jesus having only one biological parent, queries Wittgenstein, based on never knowing anyone not to have had two biological parents and on the sexual nature of persons? "But then," he asks, "is that really a proof?"[41] His querying further about the 'proof' being akin to a scientific hypothesis that is repeatedly confirmed by disbelievers, but which is no surer than confirming that yet another person has two parents, brings to mind Lukasiewicz. He might compare belief in a hypothesis to belief in the principle that specifies that something cannot have incompatible attributes.

If the principle and hypothesis were true, then it would *follow logically* that there are not the attributes and that persons have two human parents. But the reverse reasoning is actually proper: There in fact always being the parents and never the attributes would, for Lukasiewicz, be both a *reason for* the sentences being true and *real cause* of why they are true.[42] This reasoning comes closer to reflecting a genuine realism regarding 'truth' whose truth-condition is an experienced reality, not reality having norms of reason imposed on it *a priori* – notwithstanding in science that there may be other modes of reasoning to true hypotheses or theories congruent with realism, such as abduction, although the belief in their absolute and exclusive truth would still be unwarranted.

B. Conflicting Rules of Reason, Reason Conflicting with Rules

How may reason conflict with its rules? Bearing on theories construed as propositions, a rule of propositional logic holds that a false conjunct falsifies a conjunctive proposition. But one may ordinarily reason, or reason commonsensically, that ($p \wedge q$) with one false conjunct and ($r \wedge s$) with two are not equally false. What of universal claims whose falsity is denied since, while they address reality, they are

[39] Wittgenstein, *On Certainty*, 32e, #239.

[40] Christ's real presence does not exclude "other types of presence as if they could not be 'real' too," but is still "a *substantial* presence by which Christ [is] entirely present." *Catechism of the Catholic Church* (Citta del Vaticano: Libreria Editrice Vaticana, 1994), 346, #1374.

[41] Wittgenstein, *On Certainty*, 32e, #240.

[42] Logic, for Aristotle, studies the *cause* of truth. 'Truth' thereby brings to mind this point by Owen LeBlanc, "Lukasiewicz, Aristotle, and Contradiction," *Papers on Lukasiewicz*, Section 5.

deemed indubitably true? Would their truth not be empirically contingent and logically uncertain? If they were certain logically, then they would be trivial. Their not being trivial or true analytically seems as obvious as being unable to certify their truth inductively by sense experience since all things are not experienced and even if they were, how would this be known? In short, are rules about the nature of trivial and empirically contingent reasoning outweighed by other rules that bear on reasoning such as the Contradiction Principle? Should this Principle or any rule be abided by blindly? If so, what are the reasons for doing that?

Indeed, notes LeBlanc, after probing "relations between the principle of contradiction and other laws, Lukasiewicz observes that [the principle] is not in fact very useful as a logical tool, and consequently he calls into question its status as the most fundamental of all principles."[43] Do these questions about principles and rules evoke a skeptical regress, similar to that which concerned Aristotle in his *Posterior Analytics* (Bk. 1, Ch. 3), where either rules or beliefs would be justified by others and they yet by others that *ex hypothesi* are merely further beliefs and rules that beg for justification?

Contrary to this futile justification, Lukasiewicz presaged Wittgenstein who explains how "I could say… 'I have two hands is an irreversible belief'" if it expresses a refusal of metaphysical retorts to be a disproof.[44] Does this sort of disproof exhaust criticism of the Principle of Contradiction? As the belief in one's having two hands is as certain as any evidence adduced to support the belief such as looking at them (since Cartesian doubts about dreaming, for example, would be metaphysical), the Principle may seem equally inviolable without appealing to perceived objects. An uncertainty about the objects, however, can differ in terms of metaphysical doubts being *disingenuous* and *genuine* doubts having factual reasons for challenging the belief: A belief in the Principle may either be effectively questioned or permit plausible counter evidence, if not allow for a patent disproof. Lukasiewicz noted also that although a contradictory object would have an imperceptible negation, a negation could still be inferred.[45]

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[43] LeBlanc, "Lukasiewicz, Aristotle, and Contradiction," Section 9.

[44] Cf. Wittgenstein, *On Certainty*, 33e, #245.

[45] Lukasiewicz stated, "it is generally impossible to suppose that we might meet a contradiction in perception; the negation… is not at all perceptible… contradictions could only be inferred." See Lukasiewicz, "O zasadzie sprzecznosci u Arystotelesa (Über den Satz des Widerspruchs bei Aristoteles)", *Bulletin International de l'Académie des Sciences de Cracovie. Classe de Philosophie et d'Histoire* (1910), 15-38 (engl. trans.: Lukasiewicz [1971,1979]; French trans.: Lukasiewicz [1991]). Originally in Venanzio Raspa, "Lukasiewicz on the Principle of Contradiction," *Journal of Philosophical Research* 24 (1999): 57-112. From Venanzio Raspa "Lukasiewicz on the Principle of Contradiction," *Papers on Lukasiewicz*, http://www.fmag.

Inferences cast doubt on believing the Principle of Contradiction such as either light being both a wave and non-wave or all phenomena permitting inferences to logically inconsistent theories, discussed below, contrast to one's everyday belief in having two hands against which metaphysical doubts are raised rather than physical evidence. Here, Wittgenstein's distinction of resolvable *genuine doubts* by 'looking' from *metaphysical doubts* that are senseless (because anything adduced as evidence is also doubted),[46] bears on inferences noted by Lukasiewicz that render sensible a doubt about the Principle. In a single stroke, it is evidently the case both that there is no vicious regress of belief and that a belief in the Principle is dubiously founded when Wittgenstein notes, "At the foundation of well-founded belief lies belief that is not founded."[47]

Can the Principle be well founded if contradictions seem unavoidable to even present-day scientific realists?[48] These realists remind one of Lukasiewicz who said that while negations in phenomena are not seen, "contradictions could… be inferred."[49] Inferences would be *a posteriori*, not *a priori*, and be problematic only if a rationality of science is confused with a genuine scientific realism. This realism is often believed in a dogmatic sense to be rational. *The Rationality of Science* by W. H. Newton-Smith, for instance, is an admirable defense of realism.

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unict.it/~polphil/polphil/Lukas/RaspaLukas.html, 22 Dec 2007, online without page/section numbers.

[46] Doubt about the wine being blood contrasts to doubts about having two hands. Though replies by believers "I know that…" are senseless in both cases insofar as there is no way to back up the words, for Wittgenstein, doubt about the hands is senseless without any caveat since the doubt is rooted in the mundane or ordinary and there is no ordinary way to assuage it. But doubt about the blood involves a lack of faith and faith is not senseless in the sense that it would *not* be needed if God merely did what is empirically improbable; explaining why he was impatient "with attempts to give religion a *rational* foundation." See Norman Malcolm, *Ludwig Wittgenstein: A Memoir*, 2nd ed. with Wittgenstein's letters to Malcolm (New York: Oxford University Press, 1984) 59.

[47] Wittgenstein, *On Certainty*, 33e #253.

[48] These realists should recall University of Hamburg philosopher Edward Conze who, in 1935, listed a litany of thinkers in different times and cultures who denied the Principle such as Nicholas of Cusa, Hegel, Bostroem, Bradley, India's Madhyamikas (Nâgârjuna), China's Taoists, Levy-Bruhl who indicated its accepted violations by so-called 'primitive minds' and Svend Ranulf who established its contravention by Eleatic thought. Can these deviators, he asks, be dismissed "with an impatient wave of the hand?" See Conze's "The Objective Validity of the Principle of Contradiction," *Philosophy* 10, 38 (1935): 205-218.

[49] Lukasiewicz, "O zasadzie sprzecznosci u Arystotelesa (Über den Satz des Widerspruchs bei Aristoteles)", *Bulletin International de l'Académie des Sciences de Cracovie. Classe de Philosophie et d'Histoire* (1910): 15-38. From Raspa, "Lukasiewicz on the Principle of Contradiction," online without page/ section numbers.

But the nature of realism would be clarified and his case strengthened if a rationality of inferring contradictions were recognized.[50] The recognition bears on feasible solutions to other dilemmas such as an Underdetermination-of-Theory-by-Data (UTD) Thesis, discussed later. In being reminiscent of rival geometries, the Thesis allows for logically possible inferences to empirically equivalent theories that are not only inconsistent but also contradictory. And the possible contradiction is exacerbated by well-known anomalies in quantum physics such as light being both a wave and non-wave in terms of de Broglie's particle-wave equation.

Precisely, if modern formalized physics is a paradigm knowledge-yielding enterprise, but allows theories and theoretic entities to violate rules of reason, by permitting *inter alia* either contradiction or logical inconsistency, then it is easy to see why many philosophers of science may think that objective knowledge is precluded for the *less rigorous* human sciences, politics and ethics. A seriousness of this implication and value of Lukasiewicz's insights evoke an old adage: If formalized physics 'sneezes,' all other cognitive studies 'catch pneumonia' – to use metaphors for the knotty epistemological dilemmas. Thus the dilemmas posed by evident violations of the Principle of Contradiction beg for solutions that are nothing less than urgent. That this urgency bore on the logical issues at hand is illustrated by Lukasiewicz's criticism of Aristotle's excessive influence on provoking obstinate commitments to the Principle in the face of certain esoteric scientific advances in particular and intellectual developments in general.

C. Non-Euclidean Challenges to Scientific Rationality

Some of these general developments, bearing on inferences to contradictory theories and theoretic entities, include non-Euclidean Geometry. Venanzio Raspa of the Università di Urbinonotes notes that in 1918 at a farewell lecture at Warsaw University, Lukasiewicz announced his pioneering work on a three-valued logic and declared also that he had "published a book on the principle of contradiction in Aristotle's work."[51] His book on this work held that as Euclidean Geometry was wrongly thought to be axiomatically true, but resulted in non-Euclidean Geometry by

[50] W.H. Newton-Smith, *The Rationality of Science* (London: Routledge & Kegan Paul, 2007), 41-43, suggesting that rival theories T_1 and $\sim T_1$ be integrated into a more global theory T_2. But why could not $\sim T_2$ obtain, admitting of another more global theory and it of a rival theory and so on?

[51] See "Tresc wykladu pozegnalnego prof. Jana Lukasiewicza, wigloszonego w auli Uniwersytetu Warszawskiego dnia 7 marca 1918 r.," *Pro Arte et Studio* 11 (1918): 3-4 (aka "Farewell Lecture by Professor Jan Lukasiewicz, Delivered in the Warsaw University Lecture Hall on March 7, 1918"). From Raspa, "Lukasiewicz on the Principle of Contradiction," online without page numbers.

an attempted *reductio ad absurdum*, Lukasiewicz "strove to demonstrate that the principle is not so self-evident as it is believed to be" by seeking to "construct non-Aristotelian logic..."[52] And although he conceded that efforts to develop this logic had failed (perhaps precipitously), his comparison of self-evident geometric truth to the truth of Aristotle's Principle would not be lost on one prominent logician who noted that although the self-evident truth of Euclidean axioms was "long believed," it was "not believed quite whole-heartedly":

> While there was no doubt about the *truth* of Axiom 12 [parallel postulate]… its *self-evidence* was denied, which was deemed sufficient reason to relegate it… to the less dignified status of a mere theorem… The most fruitful attempt [to prove it a theorem] was that of the Italian mathematician Gerolamo Saccheri (1667-1733) who *replaced* the parallel postulate by alternative, contrary assumptions, and then sought to derive a contradiction… [But] instead of proving the parallel postulate, Saccheri (unknowingly) did something more important: he was the first to set up and develop a system of non-Euclidean geometry.[53]

Non-Euclidean geometry was developed later by Lobachevsky and Riemann, among others, and came to be accepted as being truer of real space in terms of Einstein's physics than the Euclidean geometry supposed by Newton. Yet physicist Paul Marmet notes that Einstein's space and time distortions are unneeded for various classical phenomena. These phenomena include the perihelion advance of Mercury because it is entirely explicable by Newton and mass-energy conservation.[54] And with respect to this classical domain of phenomena that do not approach the speed of light, there is also an evident epistemic impossibility of Newton's theory being wholly false when it makes *systematically* true predictions. Unless the predicted phenomena were reflected with an approximate truth by the theory, the theory's predictions would be inexplicable. And if this reasoning holds for Einstein's theory, with its relevance to near speed-of-light phenomena, a case can be made also for a truth of the inconsistent geometries that underlie the theories: Real space may have attributes of being both Euclidean and non-Euclidean.

Though non-Euclidean geometry underlies a relativistic physics that may generally apply to all that classical physics does, a greater simplicity of the latter physics may make it epistemologically preferable to the physics of Einstein via Occam's razor in the classical domain. These different domains do not exclude the

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[52] Raspa, "Lukasiewicz on the Principle of Contradiction," online without page numbers.

[53] Irving Copi, *Symbolic Logic*, 3rd ed. (London: Collier-Macmillan, 1972), 182, 183.

[54] Paul Marmet, "A Logical and Understandable Explanation to the Advance of the Perihelion of Mercury," presented to the Society for Scientific Exploration at Albuquerque 3-5 June 1999, http://www.newtonphysics.on.ca/MERCURY/Mercury.

problem of inconsistent geometries that are compounded by a unitary-invariant geometry (Fubini-Study metric) for quantum mechanics,[55] for example, but rather exacerbate the dilemma. For there may be *one* reality with *many* incompatible attributes that include the different domains and natures of geometric space, bringing to mind in addition to physics the metaphysical problem of the *One* and *Many* that was addressed theologically by Van Til.

Is Van Til supported surreptitiously by an objection that the geometric conflict is only apparent since the theories are semantically equivalent by an approach of W.V. Quine? His "semantic ascent," says Dallas Willard, purportedly eludes the theories' ontological implications by talk about "non-linguistic [ontological] matters" ascending to "entities, events, or structures that are constituents of language."[56] We supposedly ascend "to a common part of two fundamentally disparate conceptual schemes," from say "whether miles *exist* to… uses of the word 'mile'" whereby differing theories are not supposed. But we most often actually ascend, says Willard, "from philosophically contested points about what exists to… contested points about the nature and function of names." Thus he asks, "Are not Wittgenstein and Carnap, John Wisdom and Gustav Bergmann only by courtesy or confusion said to have been talking about the same thing?" and answers, "insofar as in our meta-language we are still 'speaking of objects,' Quine would be the very first to deny that semantic ascent will free us from ontological presumption."[57]

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[55] See Dorje Brody and Lane Hughston, "Geometric Quantum Mechanics," *Journal of Geometric Physics* 38 (2001): 19-53, where "the manifold of pure quantum states is a complex projective space endowed with the unitary-invariant geometry of Fubini and Study."

[56] The quotes in this paragraph are from Willard's "Why Semantic Ascent Fails," *Metaphilosophy* 14, 3/4 (1983): 276-290, from http://www.dwillard.org/articles/artview, emphasis, 27 Dec 2011.

[57] Presumptions may concern two evident contradictions to Euclidean geometry: 1) 'positively' curved space in terms of spherical/elliptical geometry and 2) 'negatively' curved space in terms of hyperbolic/Lobachevskian geometry. Though the parallel postulate concerns straight lines so that a genuine non-Euclidean geometry may not seem to be achieved, K.L. Ross distinguishes 'extrinsic' curvature from one that is 'intrinsic.' "A space can possess 'intrinsic' curvature, yet contain lines ('geodesics') that will be straight according to any… measurement intrinsic to that space. A geodesic is 'straight' in relation to its own manifold. Euclidean straightness thus characterizes the geodesic of a three dimensional space with no intrinsic curvature…" See Ross' "The Ontology and Cosmology of Non-Euclidean Geometry," http://www.friesian.com/curved-1.htm, 2011.

D. Inconsistent Theories and Theoretical Entities: The UTD Thesis

If we recall that Euclidean geometry's axiomatic truth was insisted on as inflexibly as that of the Contradiction Principle, the Principle's relevance to rival geometries and geometric truth having as its truth-condition the way reality really *is* (at least approximately) render reasonable Lukasiewicz's approach. In holding that contradictions can be inferred from phenomena by instrument-aided or naked-eye observation, his approach both reveals an authentic scientific rationality of reasoning from reality without imposing on it any norms of reason and anticipates knotty epistemic problems as well as their novel solutions. Consider a solution to light being both a wave and non-wave, for instance, after noting a Lukasiewiczian response to one of the most formidable challenges to a rationality of science: the Underdetermination-of-Theory-by-Data (UTD) Thesis.

The strongest version of this Thesis specifies the logical possibility for any set of data to admit of contradictory but empirically equivalent theories that can equally explicate, manipulate and predict phenomena in a given domain, say Newton's where Planck's constant is small and bodies do not approach the speed of light, or one applicable to light speed such as Einstein's. In being equivalent, realists could evidently not say which is true: an Einsteinian theory T_E or non-Einsteinian $\sim T_E$, a Newtonian theory T_N or non-Newtonian $\sim T_N$. Theories could be systematically underdetermined by a straightforward translation procedure that permits any data addressed by, say, T_N to be transposed into $\sim T_N$'s account.[58] This would entail an empirical equivalence of $\sim T_N$ and T_N of all known, as well as of all logically possible results. These results notwithstanding, contradictory theories could be construed as conjunctive propositions, to which 'truth' is ascribable, if the laws (L) of a given theory T_o are read as either $(L_1 \wedge L_2)$ or $[(L_1 \wedge L_2) \wedge L_3]$ and $\sim T_o$ as $\sim(L_1 \wedge L_2)$ or $\sim[(L_1 \wedge L_2) \wedge L_3]$ depending on the number of laws.

But the contradiction is possible and reflects a Lukasiewiczian realism: Reality is the truth-condition for 'truth,' not a truth of the Principle for how reality must be. Reality need not conform to the Principle and it should not be imposed on reality. Exactly, de Broglie's particle-wave equation wherein light as a wave mandates understanding light as a particle agrees with a UTD Thesis, even if the equation involves inferences to theoretical entities (construable nonetheless

[58] Cf. John Worrall, "Scientific Realism & Scientific Change," *Philosophical Quarterly* 32 (1982): 201-231, referring to Henri Poincaré's example of empirically-equivalent *contradictory* theories (223). A weaker *logically inconsistent* under-determination is illustrated by a theory T joined to a purely theoretical statement (s) with no extra empirical consequences for $T \& s$ and $T \& \sim s$ (222).

by possibly contradictory equations).[59] When it is stated in the *Proceedings of the New York Academy of Sciences* that we must still conclude that light *is* and *is not* a particle (wave), theoretical constructs are not appealed to apart from evidence.[60] Evidence is not ignored as feared by Sir Karl Popper.[61] Popper confused a top-down relativistic reasoning *from* incompatible theoretical constructs *to* phenomena, which permits contradictory truth-claims *a priori*, with bottom-up inferences *from* experimental setups *to* the constructs that allows for those claims *a posteriori*.[62]

Nor is this point restricted to light. P.K. Stanford proposes a New Induction, an 'inductive rationale' to explain typical alternatives to our best theories that are "equally well confirmed by the evidence" even if we cannot "conceive of them at the time."[63] The history of science suffers from an ability to conceive only of a few theories that are well confirmed. The rest routinely revealed alternatives that came to be "well-confirmed by the previously available evidence…"[64] Happily, evidence for this weaker Underdeterminism is said to be fallible, given his mere induction, so that many champions of the Thesis will be disappointed! The disappointed would be various antirealists with vested interests in an absence of objective truth: social constructionists, deconstructionists and multiculturalists who have endeavored to institutionalize an epistemic relativism.

Though not seeking to support relativism but rather meta-scientific pursuits of truth, John Worrall worries about a logically compelling case against realism. And although he augments Stanford's optimistic fallibility by remarking that some seemingly inconsistent theories were reconciled, say integrating data of the

[59] The particle-wave equation is $\lambda = h/p = h/mv$ where λ is wavelength, h is Planck's constant and $p = mv$ = the magnitude of a moving particle's momentum.

[60] Saul Youssef, "Is Quantum Mechanics An Exotic Probability Theory?" in *Fundamental Problems in Quantum Theory: A Conference Held in Honor of Professor John A. Wheeler*, eds. D.M Greenberger and A. Zeilinger (New York: Annals of the New York Academy of Sciences, 1995), 904.

[61] Karl Popper, *Quantum Theory and the Schism in Physics* (Lanham: Rowman & Littlefield, 1982), 142.

[62] The Greek sophist Protagoras held that "Man is the measure of what *is*." I argued elsewhere that he sometimes seems to reason *from* reality *to* contradictory ideas, a *bottom-up* reasoning that evidences, not an irrational relativism, but rather a relativistic realism that presages Lukasiewicz!

[63] See P. K. Stanford, "Refusing the Devil's Bargain: What Kind of Underdetermination Should We Take Seriously?" *Philosophy of Science* 68, 3, Supplement: Proceedings of the 2000 Biennial Meeting of the Philosophy of Science Association. Part I (Sep. 2001): S1-S12.

[64] Stanford, "Refusing the Devil's Bargain," S9.

classical wave theory of light into the corpuscular-theoretic framework,[65] he notes that the stronger UTD Thesis is not subject to fallibility or reconciliation.[66] No reconciliation is possible, as noted earlier, if there is a systematic underdeterminism of theories by a straightforward translation procedure that permits any data addressed by theory T_o to be transposed into $\sim T_o$'s account. This entails an empirical equivalence of $\sim T_o$ and T_o not only of known results but also of all logically possible results, excluding an inductive optimism via an historical record that offers only fallible evidence of an "under-determination predicament."[67]

This predicament, that is, cannot be diminished in terms of the history of science and most scientific realists would say it is fatal. The alleged fatality may seem to beg for a logical resolution, a hope of rationalistic realists. The real solution, however, involves realizing that realism does not depend on reality conforming to reason but rather on reason depending on an experienced reality. Evidence that is rooted in this reality in terms of experimental setups or otherwise may or may not result in underdetermined theories. But even if theories were inferred that are inconsistent by a 'new induction,' the induction is based on an experienced reality and not on an epistemic relativism in which the theories are accepted *a priori* for interpreting reality without evidence.

Evidence of contradictory but empirically equivalent theories is, in fact, no threat to a rational scientific realism. For the realism regards 'truth' as reflecting how reality really *is* apart from either rational or non-rational norms imposed on it, this parodying relativism! Relativism was unacceptable to Lukasiewicz. In critiquing the Contradiction Principle, he "decidedly places himself in the stream of European logical realism [wherein what is *ontologically* involved in assessing logical truths may restrain their applicability[68]]."[69]And in virtue of this

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[65] Despite Worrall's allowance for a weak UTD Thesis, Youssef's aforesaid point (*limited* to two-slit experiments) is now extended to light *at all times* having both wave and particle aspects. See Shahriar S. Afshar, Eduardo Flores, Keith F. McDonald, and Ernst Knoesel, "Paradox in Wave-Particle Duality," *Foundations of Physics* 37, 2 (2007): 295-305.

[66] Worrall, "Scientific Realism & Scientific Change," 223 (paraphrase). Stanford's *inductive optimism* to support realism ignores the *deductive pessimism* of Worrall. And Worrall's appeal to equivalent results of observation is not, as some say, undercut by observation's theory-ladeness: See Newton-Smith on 'low-level observation terms' in *The Rationality of Science*, 28, and my case for non-conceptual observation in "Physics and Phenomenology," *New Horizons in the Philosophy of Science*, ed. David Lamb (Farnham, UK: Ashgate Publishing, 1992), 66-86.

[67] Stanford, "Refusing the Devil's Bargain," S11.

[68] See E.S.G. Lombardo's review of *The Positivist and the Ontologist: Bergmann, Carnap and Logical Realism*, by Herbert Hochberg (GA: Rodopi Press, 2001), in *Mind* 112 (2003): 724–28.

[69] Raspa, "Lukasiewicz on the Principle of Contradiction," online without page numbers.

applicability being doubted by St. Augustine because the Trinity reflected our triune existence, an existence experienced incontrovertibly and phenomenologically as being both *one* and *not one* inseparable life, Lukasiewicz's allowance for contravening the Principle is not only *not* at odds with a central foundation of traditional theology but also allows logically for the Principle's violations in science as well. In sum, his position as gleaned from his own works and the works of those who knew him best indicates that the violations agree superbly with a scientific and theological realism.

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